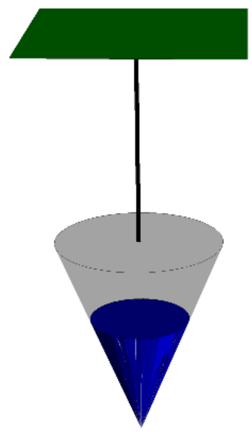
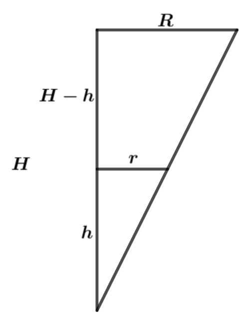
Name: *Caleb McWhorter — Solutions* **Quiz 7: Work**

MAT 296 **Fall 2017**

Problem 1: A storage tank filled with a volatile liquid with density ρ is located so the top of the tank is 10 m below the ground. The tank, shown below, is conical with top width of 14 m and height 22 m. The tank is filled to a depth of 8 m. Set up completely as possible *but do not integrate* an integral to calculate the work required to empty the contents of the tank to the surface.



Since the top of the tank is width 14 m, the 'base' radius of the cone is $R = 7$ m. The cone has height $H = 22$ m. Taking a slice through the center of the cone, we have the following diagram: By similar



triangles, we have $\frac{R}{H} = \frac{r}{h}$ so that $r = h\frac{R}{H}$ and $r^2 = h^2\frac{R^2}{H^2}$. Now consider the force on some small slice of liquid in the tank parallel to the ground: $F_i = m_i g = \rho V_i g$. The small slice is approximately a cylinder with radius r so that $F_i = \rho(\pi r_i^2 \Delta h)g$. This slice has to be moved a distance $10 + (H-h)$ m to reach the surface. Then we have $W_i = F_i d_i = (\rho \pi r_i^2 \Delta h g)(10 + H - h)$. Therefore, the total work is approximately

$$W \approx \sum_{i=1}^{n} W_i = (\rho \pi r_i^2 \Delta h\, g)(10 + H - h)$$

But then we have

$$W = \lim_{\Delta h \to 0} \sum_{i=1}^{n} W_i = \lim_{\Delta h \to 0} \sum_{i=1}^{n} (\rho \pi r_i^2 \Delta h\, g)(10 + H - h) = \int_0^8 (\rho \pi r_i^2\, g)(10 + H - h)\, dh = \frac{\rho \pi g R^2}{H^2} \int_0^8 h^2 (10 + H - h)\, dh$$