**Name:** *Caleb McWhorter — Solutions* **Quiz 9: Series**

**MAT 296** **Fall 2017**

**Problem 1:** Mark the following statements T for 'True' or F for 'False'.

(a) ___*T*___: If a series $\sum a_n$ converges, then $\lim\limits_{n\to\infty} a_n = 0$.

(b) ___*F*___: If $\lim\limits_{n\to\infty} a_n = 0$, then $\sum a_n$ converges.

(c) ___*F*___: If $\sum a_n$ diverges, then $\lim\limits_{n\to\infty} a_n \neq 0$.

(d) ___*T*___: If $\lim\limits_{n\to\infty} a_n \neq 0$, then $\sum a_n$ diverges.

(e) ___*T*___: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

**Problem 2:** Determine whether the following series converges or diverges. If the series converges, prove it. If the series diverges, prove it.

$$\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$$

$$\lim_{n\to\infty} \cos\left(\frac{1}{n}\right) = \cos 0 = 1 \neq 0$$

*Therefore, the series* $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$ *diverges by the Divergence Test.*

**Problem 3:** Determine whether the following series converges or diverges. If the series converges, find the sum. If the series diverges, prove it.

$$\sum_{n=2}^{\infty} \frac{5\pi(3^{n-1})}{2^{2n+1}}$$

$$\sum_{n=2}^{\infty} \frac{5\pi(3^{n-1})}{2^{2n+1}} = \sum_{n=2}^{\infty} 5\pi \frac{3^{n-1}}{2^{2n+1}} = \sum_{n=2}^{\infty} 5\pi \frac{3^{n} \cdot 3^{-1}}{2^{2n} \cdot 2} = \sum_{n=2}^{\infty} \frac{5\pi}{6} \cdot \frac{3^{n}}{4^{n}} = \sum_{n=2}^{\infty} \frac{5\pi}{6} \cdot \left(\frac{3}{4}\right)^{n}$$

*This is a geometric series with $r = 3/4$. Since $|r| < 1$, the series converges.*

$$\sum_{n=2}^{\infty} \frac{5\pi}{6} \cdot \left(\frac{3}{4}\right)^{n} = \frac{\frac{5\pi}{6} \cdot \frac{3^2}{4^2}}{1 - \frac{3}{4}} = \frac{5\pi}{6} \cdot \frac{3^2}{4^2} \cdot \frac{4}{1} = \frac{5\pi}{2} \cdot \frac{3}{4} \cdot 1 = \frac{15\pi}{8}$$