Lecture 10: Improper Integrals

Problem 1: Decide whether the following (improper) integrals converge or diverge. If the integral converges, give its value.

(a) $\displaystyle\int_1^\infty \frac{dx}{x^4}$

(b) $\displaystyle\int_5^\infty \frac{dx}{x^3}$

(c) $\displaystyle\int_0^3 \frac{dx}{x}$

(d) $\displaystyle\int_0^\infty \frac{dx}{\sqrt{x^3}}$

(e) $\displaystyle\int_0^\infty \cos(\pi x)\,dx$

(f) $\displaystyle\int_{-\infty}^0 x^2 e^x\,dx$

(g) $\displaystyle\int_{-\infty}^\infty \sin x\,dx$

(h) $\displaystyle\int_0^\infty x e^{-x/3}\,dx$

(i) $\displaystyle\int_{-\infty} 2^x\,dx$

(j) $\displaystyle\int_{-\infty}^\infty \arctan x\,dx$

(k) $\displaystyle\int_0^\infty e^{-x}\sin x\,dx$

(l) $\displaystyle\int_{-\infty}^\infty \frac{dx}{x^2+9}$

(m) $\displaystyle\int_0^\infty \frac{e^x}{e^x+1}\,dx$

(n) $\displaystyle\int_0^1 x\ln x\,dx$

(o) $\displaystyle\int_0^{\pi/2} \tan x\,dx$

(p) $\displaystyle\int_0^3 \frac{dx}{9-x^2}$

(q) $\displaystyle\int_1^3 \frac{5}{(x-2)^{8/3}}\,dx$

(r) $\displaystyle\int_0^1 \frac{dx}{\sqrt[3]{x-1}}$

Problem 2: Decide whether the following (improper) integrals converge or diverge. If the integral converges, give its value.

(a) $\displaystyle\int_1^\infty \frac{\cos^2 x}{x^2}\,dx$

(b) $\displaystyle\int_0^\infty \frac{dx}{x+e^x}$

(c) $\displaystyle\int_1^\infty \frac{17+5\sin^3(4x)}{\sqrt{x}}\,dx$

(d) $\displaystyle\int_1^\infty \frac{e^{-x}}{x}\,dx$

(e) $\displaystyle\int_0^\infty e^{-x^2}\,dx$

(f) $\displaystyle\int_0^\infty \frac{x}{x^3+1}\,dx$

Problem 3: Determine whether the following statements are true or false. Explain your reasoning.

(a) If $f(x)$ is continuous on $[0,\infty)$ and $\lim\limits_{x\to\infty} f(x) = 0$, then $\int_0^\infty f(x)\,dx$ converges.

(b) If $f(x)$ is continuous on $[0,\infty)$ and $\int_0^\infty f(x)\,dx$ diverges, then $\lim\limits_{x\to\infty} f(x) \neq 0$.

(c) If $f(x)$ is continuous everywhere and $f(-x) = -f(x)$, then $\int_{-\infty}^\infty f(x)\,dx = 0$.

(d) If $f(x)$ is continuous everywhere and $f(x) = f(-x)$, then $\int_0^\infty f(x)\,dx$ exists if and only if $\int_{-\infty}^\infty f(x)\,dx$.