Problem 1:

(a) Explain what a sequence is.

(b) Explain what it means for a sequence to converge.

(c) Say that it means for $\lim_{n\to\infty} a_n = 57$.

(d) What does it mean for a sequence to diverge?

(e) In what ways can a sequence diverge?

Problem 2: Find the first 5 terms of $x_n = \dfrac{n}{n+1}$. Does the sequence converge? If so, show that the limit is what you claim. If it diverges, explain why.

Problem 3: If a_n is a sequence with $a_0 = 5$ and $\lim_{n\to\infty} a_n = -4$, does the limit change if we instead change $a_0 = 5$ to $a_0 = 4$? Explain.

Problem 4: Write the first 7 terms of $\left\{\cos\left(\dfrac{n\pi}{4}\right)\right\}$. Does the sequence converge? Explain.

Problem 5: Find a formula for the following sequences:

(a) $\{1, 2, 4, 6, 8, 10, \ldots\}$

(b) $\{4, 7, 10, 13, 16, \ldots\}$

(c) $\{1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, \ldots\}$

(d) $\{2, 3, \frac{9}{2}, \frac{27}{4}, \frac{81}{8}, \frac{243}{16}, \ldots\}$

Problem 6: A sequence is given by $a_0 = 2$ and $a_{n+1} = 2a_n - 1$. Find a_6.

Problem 7: A sequence is given by $b_1 = 1$ and $b_{n+1} = 3b_n + 2$. Find b_5.

Problem 8: A sequence is given by $x_0 = 1$, $x_1 = 3$, and $x_{n+2} = 4x_n - x_{n-1}$. Find x_4.

Problem 9: Determine whether the following sequences converge or diverge. If the sequence converges, find the limit. Be sure to justify the limit.

(a) $a_n = \left(\dfrac{1}{2}\right)^n$

(b) $b_n = 2^n$

(c) $c_n = 5^{-n}$

(d) $d_n = \tan^{-1} n$

(e) $x_n = \dfrac{2n^2+1}{150n+65}$

(f) $y_n = \dfrac{3n+1}{2n+7}$

(g) $z_n = \dfrac{5n+1}{n^2+3}$

(h) $s_n = \dfrac{4^{n+2}}{5^n}$

(i) $t_n = (-1)^n$

(j) $a_n = \left(1 + \dfrac{3}{n}\right)^n$

(k) $b_n = \dfrac{\arctan n}{n}$

(l) $r_n = \tan\left(\dfrac{4n\pi}{1+12n}\right)$

(m) $s_n = \dfrac{2^n}{n!}$

(n) $t_n = \dfrac{\cos n}{\sqrt{1+n}}$

(o) $u_n = \dfrac{\ln n}{n}$

Problem 10: Find the limit of the sequence

$$\left\{\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \cdots\right\}.$$

Problem 11: Find the limit of the sequence

$$\left\{\sqrt[3]{3}, \sqrt[3]{3\sqrt[3]{3}}, \sqrt[3]{3\sqrt[3]{3\sqrt[3]{3}}}, \cdots\right\}.$$

Problem 12: If $\lim_{n\to\infty} a_n = 0$ and $\{b_n\}$ is bounded, is $\lim_{n\to\infty}(a_n b_n) = 0$? Explain your reasoning.

Problem 13: What does it mean to say $\displaystyle\sum_{n=1}^{\infty} a_n = 10$? Explain.

Problem 14: Explain some of the ways that a series can diverge. Give examples to demonstrate your statements.

Problem 15: Find $\displaystyle\sum_{i=1}^{4}(i^2 - 1)$.

Problem 16: Find $\displaystyle\sum_{n=1}^{5} \frac{1}{n(n+1)}$.

Problem 17: Express the following numbers as a fraction:

(a) $0.\overline{5}$

(b) $1.\overline{4}$

(c) $2.6\overline{7}$

(d) $0.\overline{55}$

(e) $5.123123\overline{123}$

Problem 18: Determine to which of the following series, if any, the Divergence Test applies

(a) $\sum_{n=1}^{\infty}(-1)^n$

(b) $\sum_{n=1}^{\infty}\frac{n+1}{n^2+1}$

(c) $\sum_{n=1}^{\infty}\frac{2n+3}{1-5n}$

(d) $\sum_{n=1}^{\infty}\sqrt[n]{2}$

(e) $\sum_{n=1}^{\infty}\arctan n$

(f) $\sum_{n=1}^{\infty}\cos\left(\frac{1}{n}\right)$

(g) $\sum_{n=1}^{\infty}n\sin\left(\frac{1}{n}\right)$

(h) $\sum_{n=1}^{\infty}(\sin 1)^n$

(i) $\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$

(j) $\sum_{n=1}^{\infty}\frac{1+4^n}{5^n}$