

**Problem 1:** Geometric Series: Determine if the following series diverge or converge. If the series converges, find the sum.

(a)  $1 + \frac{1}{4} + \frac{1}{16} + \frac{64}{+} \dots$

(h)  $\sum_{n=1}^{\infty} \frac{5^n}{6^{n-1}}$

(b)  $2 - 3 + \frac{9}{2} - \frac{27}{4} + \frac{81}{8} - \dots$

(i)  $\sum_{n=0}^{\infty} \frac{15}{(-4)^{n+1}}$

(c)  $5 + 2 + \frac{4}{5} + \frac{8}{25} + \frac{16}{125} + \dots$

(j)  $\sum_{n=2}^{\infty} \frac{4}{9} \left(\frac{1}{9}\right)^n$

(d)  $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{9} + \frac{1}{8} + \frac{1}{27} + \dots$

(k)  $\sum_{n=0}^{\infty} \frac{1 + 2^n}{5^n}$

(e)  $0.02 + 0.0004 + 0.000008 + \dots$

(f)  $\sum_{n=0}^{\infty} \frac{2}{5^{n+1}}$

(l)  $\sum_{n=1}^{\infty} \frac{5 + 4^{n-1}}{3^n}$

(g)  $\sum_{n=1}^{\infty} \frac{4}{(-3)^{n-1}}$

(m)  $\sum_{n=0}^{\infty} \frac{1}{3} \left(\frac{2}{5}\right)^{2n+1}$

**Problem 2:** Telescoping Series: Determine if the following series diverge or converge. If the series converges, find the sum.

(a)  $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$

(e)  $\sum_{n=2}^{\infty} \frac{3}{(n-1)(n+2)}$

(b)  $\sum_{n=1}^{\infty} \frac{2}{n^2 + 2n}$

(f)  $\sum_{n=1}^{\infty} \ln\left(\frac{n+2}{n}\right)$

(c)  $\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$

(g)  $\sum_{n=1}^{\infty} \ln\left(\frac{1}{n}\right)$

(d)  $\sum_{n=1}^{\infty} \frac{1}{2n(n+1)}$

(h)  $\sum_{n=1}^{\infty} \frac{1}{(2n+1)(2n+3)}$

**Problem 3:** Find two divergent series  $\sum a_n$ ,  $\sum b_n$  such that  $\sum a_n + b_n$  converges.

**Problem 4:** If  $\sum a_n$  converges, does  $\sum \frac{1}{a_n}$  converge or diverge?

**Problem 5:** True or False: If  $\lim a_n = 0$ , then  $\sum a_n$  converges.

**Problem 6:** If  $a_n$  is the  $n$ th term of the Fibonacci sequence ( $n \geq 1$ ), show that

$$\sum_{n=0}^{\infty} \frac{1}{a_{n+1}a_{n+3}} = 1$$

[Hint: Show that  $\frac{1}{a_{n+1}a_{n+3}} = \frac{1}{a_{n+1}a_{n+2}} - \frac{1}{a_{n+2}a_{n+3}}$ ]

**Problem 7:** For each of the following series, find all values for  $x$  for which the series converges.

(a)  $\sum_{n=1}^{\infty} \frac{x^n}{3^n}$

(d)  $\sum_{n=1}^{\infty} \frac{5}{6} \left( \frac{x-2}{5} \right)^n$

(b)  $\sum_{n=1}^{\infty} (2x)^n$

(e)  $\sum_{n=1}^{\infty} (-1)^n x^{2n}$

(c)  $\sum_{n=1}^{\infty} (x+3)^n$

(f)  $\sum_{n=1}^{\infty} \left( \frac{x^2}{x^2+4} \right)^n$