Problem 1: Integral Test: Determine whether the following series converge or diverge. Be sure to fully justify your answer.

(a) $\sum_{n=1}^{\infty} \frac{1}{n+5}$

(b) $\sum_{n=1}^{\infty} e^{-n}$

(c) $\sum_{n=1}^{\infty} \frac{4}{2n+5}$

(d) $\sum_{n=2}^{\infty} \frac{1}{1-n}$

(e) $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

(f) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

(g) $\sum_{n=1}^{\infty} \frac{1}{(5n+3)^3}$

(h) $\sum_{n=1}^{\infty} \frac{n}{n^4+1}$

(i) $\sum_{n=1}^{\infty} \frac{\arctan n}{2n^2+2}$

(j) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+5}}$

(k) $\sum_{n=1}^{\infty} \frac{3}{\sqrt{n}(2+\sqrt{n})}$

(l) $\sum_{n=1}^{\infty} \frac{3}{\sqrt[5]{n^3}}$

Problem 2: Determine if the following p–tests converge or diverge.

(a) $\sum_{n=1}^{\infty} \frac{1}{n^{0.98}}$

(b) $\sum_{n=1}^{\infty} \frac{1}{n^{1.001}}$

(c) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2}}$

(d) $\sum_{n=1}^{\infty} \frac{5}{\sqrt[6]{n^7}}$

(e) $\sum_{n=1}^{\infty} \frac{6}{\sqrt[5]{n^6}}$

(f) $\sum_{n=1}^{\infty} n^{-\pi}$

(g) $\sum_{n=1}^{\infty} \frac{4}{\sqrt{n^\pi}}$

(h) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[8]{n^\pi}}$

(i) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

(j) $\sum_{n=1}^{\infty} \frac{n+1}{\sqrt{n^5}}$

Problem 3: The p–test does not *directly* imply the convergence/divergence of the following series. However, the 'p–test' can give intuition to whether the following series converge/diverge. Determine whether the following series converge/diverge.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+n+4}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n-3}{n+6}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^2+3}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^3+3}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^\pi+1}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{2+\sin n}{n}$

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{2+\sin n}{n^3}$

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^\pi+1}}$

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-n-1}{n^4+n+6}$

(j) $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{2n^3+n-1}$

Problem 4: Suppose $f(x)$ is a function to which the Integral Test applies. Let $\{a_n\}_{n=1}^{\infty}$ be the series obtained by $a_n = f(n)$ and $S_N = \displaystyle\sum_{n=1}^{N} a_n$. Suppose the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges to a number S. Show that the sum of the remaining terms, R, of the series (that is, $S - S_n$) is bounded by

$$0 \le R \le \int_N^\infty f(x)\, dx$$

Thus, we can write

$$\sum_{n=1}^{N} a_N \le \sum_{n=1}^{\infty} a_n \le \sum_{n=1}^{N} a_n + \int_N^\infty f(x)\, dx$$

[Hint: A picture like the one from 'proving' the Integral Test should help.]

Problem 5: Use the previous problem to approximate the following summations to 5 digits of accuracy:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

(c) $\displaystyle\sum_{n=1}^{\infty} e^{-n}$