**Problem 1:** Derive directly the Taylor Series for the following functions at the given center $x = c$. Find the interval and radius convergence of the power series you find.

(a) $f(x) = x^3 - x^2 + x - 1$; $x = 1$

(b) $f(x) = \sin x$; $x = 0$

(c) $f(x) = \frac{1}{x}$; $x = -2$

(d) $f(x) = \cos x$; $x = \pi$

(e) $f(x) = e^{2x}$; $x = 0$

(f) $f(x) = e^{2x}$; $x = 1$

(g) $f(x) = \cos x$; $x = \dfrac{\pi}{4}$

(h) $f(x) = \ln(x^2 + 1)$; $x = 0$

**Problem 2:** Find, using any method, the Taylor Series for the following functions at the given center $x = c$. Find the interval and radius of convergence of the power series you find.

(a) $f(x) = e^{-2x}$; $x = 0$

(b) $f(x) = x \sin x$; $x = 0$

(c) $f(x) = \dfrac{1}{x}$; $x = 3$

(d) $f(x) = \dfrac{\ln(1+x)}{x}$; $x = 0$

(e) $f(x) = 2\sin x^3$; $x = 1$

(f) $f(x) = \dfrac{1}{1-3x}$; $x = 0$

(g) $f(x) = e^{3x}$; $x = -2$

(h) $f(x) = \dfrac{2}{5+3x}$; $x = 0$

(i) $f(x) = \dfrac{x - \tan^{-1} x}{x^3}$; $x = 0$

(j) $f(x) = \dfrac{1}{x^2+1}$; $x = 0$

(k) $f(x) = \dfrac{1}{1-x}$; $x = 4$

(l) $f(x) = \dfrac{x}{2-x}$; $x = 0$

(m) $f(x) = \dfrac{x}{2-x}$; $x = 5$

(n) $f(x) = \dfrac{x^2}{(1-2x)^2}$; $x = 0$

**Problem 3:** Find a power series which converges only for $x \in [1, 3)$.

**Problem 4:** Use Taylor Series to evaluate the following:

(a) $\displaystyle\int \cos x^2 \, dx$

(b) $\displaystyle\int \frac{\ln(1-x)}{x} \, dx$

(c) $\displaystyle\lim_{x\to 0} \frac{1-\cos x}{1+x-e^x}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{n!}$

(e) $\displaystyle\lim_{x\to 0} \frac{\ln(x+1)}{x}$

(f) $\displaystyle\lim_{x\to 0} \frac{\sin x}{x}$

(g) $1 - \ln 2 + \dfrac{(\ln 2)^2}{2!} - \dfrac{(\ln 2)^3}{3!} + \cdots$

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}$

(i) $\displaystyle\int \frac{dx}{x^4+1}$

(j) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}$

(k) $\displaystyle\sum_{n=1}^{\infty}\frac{n^2}{2^n}$

**Problem 5:** Approximate the following:

1. $\arctan 0.5$
2. $\ln 1.1$
3. $\sin \frac{\pi}{4.5}$
4. $e^{0.1}$
5. $\sqrt{2.8}$