Problem 1: Perform the following integrations 'in your head.'

(a) $\int \cos(5x)\, dx$

(b) $\int \sin(3x)\, dx$

(c) $\int \frac{dx}{1-x}$

(d) $\int e^{7x}\, dx$

(e) $\int (3x+4)^{10}\, dx$

(f) $\int \sec^2(-6x)\, dx$

(g) $\int \frac{dx}{2x+1}$

(h) $\int e^{\pi x}\, dx$

(i) $\int \sqrt{1-5x}\, dx$

(j) $\int \cos(1-x)\, dx$

(k) $\int (\sqrt{\pi}x + \sqrt[3]{2})^4\, dx$

(l) $\int \csc^2(2x+3)\, dx$

(m) $\int \tan(5x)\sec(5x)\, dx$

(n) $\int 2^{7-\pi x}\, dx$

Problem 2: Integrate the following:

(a) $\int e^{\sin x}\cos x\, dx$

(b) $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx$

(c) $\int 5x^2\ln(3x^3+5)\, dx$

(d) $\int \sqrt[3]{\pi}\sin x\sin(\cos(x))\, dx$

(e) $\int \frac{3\sin(\log(x))}{5x}\, dx$

(f) $\int 5x\sqrt{1-x^2}\, dx$

(g) $\int \sec^2 x\sqrt[5]{\tan^2 x}\, dx$

(h) $\int \frac{\pi x}{(x^2+1)\sqrt[4]{\log(x^2+1)}}\, dx$

Problem 3: Calculate the following:

(a) $\int_0^{\pi/4} \cos 2x\, dx$

(b) $\int_0^8 \frac{5x}{\sqrt{x^2+36}}\, dx$

(c) $\int_1^e \frac{1-\ln x}{x}\, dx$

(d) $\int_0^{\pi} \sin^2 x\cos x\, dx$

(e) $\int_0^1 xe^{-x^2}\, dx$

(f) $\int_0^2 \sqrt{4x+1}\, dx$

Problem 4: Can 'traditional' u-substitution be used to integrate $\int x^2 e^{x^3}\, dx$? Explain.

Problem 5: Can 'traditional' u-substitution be used to integrate $\int x^2 e^{x^5}\, dx$? Explain.

Problem 6:

(a) Show the following:

$$\int f(x)f'(x)\, dx = \frac{f(x)^2}{2} + C$$

(b) Use (a) to integrate the following:

$$\int \left(x^3 e^{\sin x}\right)\left(3x^2 e^{\sin x} + x^3 e^{\sin x} \cos x\right)\, dx$$

(c) Extend (a) and show for $n \neq -1$

$$\int f(x)^n f'(x)\, dx = \frac{f(x)^{n+1}}{n+1} + C$$

Problem 7: Some integrals 'look' like they might be a u-substitution but are really just a matter of basic algebra. Integrate the following by dividing the numerator by the denominator then integrating:

(a) $\displaystyle\int \frac{x^2 + x}{x}\, dx$

(b) $\displaystyle\int \frac{5x^5 + x^2}{x}\, dx$

(c) $\displaystyle\int \frac{x^3 - 4x + 1}{x}\, dx$

(d) $\displaystyle\int \frac{\sqrt[3]{x} - x + 6}{\sqrt{x}}\, dx$

Problem 8: Some integrals are u-substitution but are 'in disguise': $\displaystyle\int \frac{1}{1 + e^x}\, dx$. [Hint: Add and subtract e^x in the numerator.]

Problem 9: Integrate $\int \tan^2(2x)\, dx$. [Hint: A trig identity 'coming from' $\sin^2 x + \cos^2 x = 1$ might help.]

Problem 10: Integrate $\int \cot x \ln(\sin x)\, dx$.