

Problem 1: Perform the following integrations ‘in your head.’

(a) $\int \cos(3x) dx$

(f) $\int \sec^2(\pi^2 x) dx$

(b) $\int \cos\left(\frac{x}{2}\right) dx$

(g) $\int \tan(5x) \sec(5x) dx$

(c) $\int \cos\left(\frac{4}{7}x\right) dx$

(h) $\int (5x + 3)^{10} dx$

(d) $\int e^{2/3x} dx$

(i) $\int \sqrt{3x + 8} dx$

(e) $\int \frac{5}{2 - 3x} dx$

(j) $\int \sqrt{(3x + 8)^7} dx$

Problem 2: Integrate the following:

(a) $\int \frac{x}{x - 6} dx$

(e) $\int x^2(1 - x)^{12} dx$

(b) $\int \frac{3x}{x + 1} dx$

(f) $\int x\sqrt{3x - 5} dx$

(c) $\int \frac{2x^2}{x + 3} dx$

(g) $\int x^2\sqrt{x + 6} dx$

(d) $\int x(2x + 1)^{95} dx$

Problem 3: Integrate the following:

(a) $\int \frac{dx}{1 + x^2}$

(e) $\int \frac{dx}{x^2 + 4}$

(b) $\int \frac{-2}{x^2 + 1} dx$

(f) $\int \frac{3}{4x^2 + 25} dx$

(c) $\int \frac{dx}{6x^2 + 6}$

(d) $\int \frac{dx}{9x^2 + 1}$

(g) $\int \frac{dx}{5x^2 + 4}$

Problem 4: The following integrals can be found on a table of integrals:

$$\int \frac{x^2}{\sqrt{ax+b}} dx = \frac{2(3a^2x^2 - 4abx + 8b^2)}{15a^3} \sqrt{ax+b}$$

$$\int \frac{dx}{x^3(x^4+a^4)} = \frac{-1}{2a^4x^2} - \frac{1}{2a^6} \tan^{-1}\left(\frac{x^2}{a^2}\right)$$

$$\int \frac{dx}{x(x^3+a^3)} = \frac{1}{3a^3} \ln\left(\frac{x^3}{x^3+a^3}\right)$$

$$\int \frac{dx}{x\sqrt{x^n-a^n}} = \frac{2}{n\sqrt{a^n}} \cos^{-1}\sqrt{\frac{a^n}{x^n}}$$

Use these formulas to integrate the following:

- (a) $\int \frac{e^{3x}}{\sqrt{e^x+3}} dx$
- (b) $\int \frac{\ln 2}{1+2^{3x}} dx$
- (c) $\int \frac{\ln 2}{\sqrt{2^{5x}-32}} dx$
- (d) $\int \frac{\cot x \csc^2 x}{\tan^4 x + 1} dx$

Problem 5: The following can be found on a table of integrals:

$$\int \frac{dx}{x^3(x^4-a^4)} = \frac{1}{2a^4x^2} + \frac{1}{4a^6} \ln\left(\frac{x^2-a^2}{x^2+a^2}\right)$$

$$\int \frac{dx}{x\sqrt{x^n-a^n}} = \frac{2}{n\sqrt{a^n}} \cos^{-1}\sqrt{\frac{a^n}{x^n}}$$

$$\int \frac{dx}{1-\cos ax} = \frac{-1}{a} \cot\left(\frac{ax}{2}\right)$$

$$\int x \cot^{-1}\left(\frac{x}{a}\right) dx = \frac{(x^2+a^2)}{2} \cot^{-1}\left(\frac{x}{a}\right) + \frac{ax}{2}$$

Use these to integrate the following:

- (a) $\int \frac{\ln 2}{\sqrt{2^{5x}-32}} dx$
- (b) $\int \frac{dx}{\sec x(1-\cos(2\sin x))}$
- (c) $\int \frac{e^{-2x}}{e^{4x}-1} dx$
- (d) $\int \frac{\ln 2 \operatorname{arccot}(2^{x-1})}{2^{-2x}} dx$