

Problem 1: Perform the following integrations:

(a) $\int x e^{-4x} dx$

(h) $\int (\ln x)^2 dx$

(b) $\int x^5 \ln x dx$

(i) $\int x \arctan x dx$

(c) $\int 3x \sin(2x) dx$

(j) $\int \left(\frac{\ln x}{x}\right)^2 dx$

(d) $\int (x^2 - \pi)e^{2x} dx$

(k) $\int x^7 \sqrt{2x^4 + 1} dx$

(e) $\int \arctan(1/x) dx$

(l) $\int \frac{x^3}{(x^2 + 5)^2} dx$

(f) $\int \ln x dx$

(m) $\int \frac{x^3 e^{x^2}}{(x^2 + 1)^2} dx$

(g) $\int \sin^{-1} x dx$

(n) $\int \cos x \ln(\sin x) dx$

Problem 2: Integrate the following functions:

(a) $\int x^3 e^x dx$

(d) $\int (3x^3 + 2x + 1) \sin(2x) dx$

(b) $\int x^3 \cos x dx$

(e) $\int x^3 e^{x/2} dx$

(c) $\int x^5 e^{2x} dx$

(f) $\int \frac{x^5}{120} \sin(x/2) dx$

Problem 3: Calculate the following:

(a) $\int e^x \cos x dx$

(d) $\int e^x \cos(x/2) dx$

(b) $\int e^{2x} \sin x dx$

(e) $\int e^{3x} \sin(4x) dx$

(c) $\int \sin(2x) \cos(3x) dx$

(f) $\int \sin(2x) \cos(x/5) dx$

Problem 4: Compute the following definite integrals:

(a) $\int_1^9 \frac{\ln x}{\sqrt{x}} dx$

(d) $\int_1^3 (\log x)^2 dx$

(b) $\int_0^2 x \sin(\pi x) dx$

(e) $\int_0^1 \sin^{-1} x dx$

(c) $\int_0^1 (2x^2 + 3)e^{-x} dx$

(f) $\int_1^9 e^{\sqrt{x}} dx$

Problem 5: Show that for $n \geq 0$,

$$\int (\ln x)^n dx = x(\ln x)^n - n \int (\ln x)^{n-1} dx$$

Problem 6: Show that for $n \geq 1$,

$$\int \sin^n x dx = -\frac{1}{n} \cos x \sin^{n-1} x + \frac{n-1}{n} \int \sin^{n-2} x dx$$

Then use this to show for odd powers, we have

$$\int_0^{\pi/2} \sin^{2n+1} x dx = \frac{2 \cdot 4 \cdot 6 \cdots 2n}{3 \cdot 5 \cdot 7 \cdots (2n+1)} := \frac{(2n)!!}{(2n+1)!!}$$

and for even powers

$$\int_0^{\pi/2} \sin^{2n} x dx = \frac{\pi}{2} \cdot \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots 2n} = \frac{\pi}{2} \cdot \frac{(2n-1)!!}{(2n)!!}$$