Problem 1: Perform the following integrations:

(a) $\int \sin^2 x \, dx$

(b) $\int \cos^2 x \, dx$

(c) $\int \sin(2x) \tan x \, dx$

(d) $\int \sin^3 x \cos^4 x \, dx$

(e) $\int \sin^6 x \cos^3 x \, dx$

(f) $\int \frac{\cos x}{\sec x} \, dx$

(g) $\int \sin x \tan^2 x \, dx$

(h) $\int \sin x \sec^2 x \, dx$

(i) $\int \sec^6 x \tan^4 x \, dx$

(j) $\int \sec^2 x \tan^3 x \, dx$

(k) $\int \tan^6 x \, dx$

(l) $\int \sin^4 x \, dx$

(m) $\int \sec(3x) \, dx$

(n) $\int \cos^3(x/3) \, dx$

(o) $\int \sin^2(5x) \, dx$

(p) $\int \sin^7(3x) \cos(3x) \, dx$

Problem 2: Perform the following integrations:

(a) $\int \frac{\cos x \sec x}{\csc x} \, dx$

(b) $\int x^2 \sin^2 x \, dx$

(c) $\int \frac{\tan^3 x}{\sqrt{\sec x}} \, dx$

(d) $\int \frac{\tan x}{\sec x} \sin^2 x \, dx$

(e) $\int \frac{\tan^2 x}{\sec^5 x} \, dx$

(f) $\int (\sin^2 x + 3) \, dx$

(g) $\int \frac{\sec x}{\tan^2 x} \, dx$

(h) $\int \frac{\sin x}{\csc x} \, dx$

Problem 3: Calculate the following definite integrals:

(a) $\int_0^{\pi/4} \tan^4 x \, dx$

(b) $\int_0^{\pi/2} \cos^4 x \, dx$

(c) $\int_0^{\pi/4} \sin x \cos x \, dx$

(d) $\int_0^{\pi} \sin^2 x \cos^3 x \, dx$

(e) $\int_0^{\pi/3} \sin x \sin(\pi \cos x) \, dx$

(f) $\int_{7\pi/6}^{5\pi/4} \sec^4 x \tan^3 x \, dx$

$$\begin{aligned}
\sin mx \sin nx &= \frac{1}{2}(\cos[(m-n)x] - \cos[(m+n)x]) \\
\sin mx \cos nx &= \frac{1}{2}(\sin[(m-n)x] + \sin[(m+n)x]) \\
\cos mx \cos nx &= \frac{1}{2}(\cos[(m-n)x] + \cos[(m+n)x])
\end{aligned}$$

Problem 4: Use the formulas above to integrate the following. Verify your answer by using integration by parts.

(a) $\displaystyle\int \sin 3x \cos 4x \; dx$

(b) $\displaystyle\int \sin 2x \sin x \; dx$

(c) $\displaystyle\int \cos 6x \cos 3x \; dx$

Problem 5: Show that

$$\int \cos^n x \; dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \; dx$$

Problem 6: Use the previous exercise to show if n is odd and at least 3,

$$\int_0^{\pi/2} \cos^n x \; dx = \left(\frac{2}{3}\right)\left(\frac{4}{5}\right)\left(\frac{6}{7}\right)\cdots\left(\frac{n-1}{n}\right)$$