Arc Length

Set up the integrals to calculate – but do not evaluate – the following lengths:

(i) The length of the curve $y^2 = x$ from $(0,0)$ to $(1,1)$

(ii) The length of the curve $y = \sin x$ from $(0,0)$ to $(2\pi, 0)$

(iii) The length of the curve $xy = 1$ from $(1,1)$ to $(2, 1/2)$

(iv) The length of the curve $x^2 + y^2 = 9$ from $(3,0)$ to $(0,-3)$

(v) The length of the curve $y^2 = 4(x+4)^3$ for $x \in [0,2]$ and $y > 0$.

(vi) The length of the curve $x = y^2 + 3$ from $(4,-1)$ to $(4,1)$

(vii) The length of the curve

$$\begin{cases} x(t) = t^3 + 2t + 1 \\ y(t) = 2t + 1 \end{cases}$$

from $t = 0$ to $t = 1$

(viii) The length of the curve

$$\begin{cases} x(t) = 2\cos t \\ y(t) = 3\sin t \end{cases}$$

for $t = 0$ to $t = 2\pi$.

(ix) The length of the curve

$$\begin{cases} x(t) = e^{-t} \\ y(t) = 2t\sin t \end{cases}$$

for $t = 0$ to $t = 3\pi$.

Area between Curves

Find the area between the given curves:

(i) $f(x) = x^2$, $g(x) = 0$, $x = -2$, $x = 2$

(ii) $f(x) = x$, $g(x) = \sin x$

(iii) $f(x) = x^2 - 10$, $g(x) = 5 - 2x$

(iv) $f(x) = \sin x$, $g(x) = \cos x$, $x = 0$, $x = 2\pi$

(v) $f(x) = \sqrt{x}$, $g(x) = x$

(vi) $f(x) = x^2$, $g(x) = \sqrt[3]{x}$

(vii) $f(x) = x^3 - 10x + 3$, $g(x) = 3 - 3x^2$

(viii) $f(x) = x^2$, $y = 4$, $x = 0$

(ix) $f(x) = \sqrt[5]{x}$, $x = 0$, $y = 32$

(x) $y = x - 1$, $y^2 = 2x + 6$

(xi) $x = y^2 - 4$, $x = y + 2$

(xii) $x = y^3 - 10y + 3$, $x = 3 - 3y^2$

Volume of Rotations

Set up the integral for calculating—but do not compute—the volume of rotating the given region about the given axis. Whenever possible, set up the integral for both the Shell Method and the Disk/Washer Method.

(i) $y = 2x - 4$, $x = 6$, $y = 0$ about the x-axis

(ii) $y = 2x - 4$, $x = 6$, $y = 0$ about the y-axis

(iii) $y = x^2$, $y = 3$, $x = 0$ about the x-axis

(iv) $y = x^2$, $y = 3$, $x = 0$, about the y-axis

(v) $y = \sqrt[3]{x}$, $y = x/4$, about the x-axis

(vi) $y = \sqrt[3]{x}$, $y = x/4$, about the line $x = 6$

(vii) $y = \sqrt[3]{x}$, $y = x/4$, about the line $x = -5$

(viii) $y = \sqrt{x-1}$, $y = (x-1)^2$ about the line $y = 7$

(ix) $y = \sqrt{x-1}$, $y = (x-1)^2$ about the line $y = -5$

(x) $y = \sqrt{x-1}$, $y = (x-1)^2$ about the line $x = 7$

(xi) $y = \sqrt{x-1}$, $y = (x-1)^2$ about the line $x = -4$

Volume by Cross Sections

(i) The base of a solid has boundary given by the curves $f(x) = x^2 - 1$ and $g(x) = 1 - x^2$. The cross sections perpendicular to the x-axis are equilateral triangles. Find the volume of the solid.

(ii) The base of a solid has boundary given by the curves $f(x) = x^2 - 1$ and $g(x) = 1 - x^2$. The cross sections perpendicular to the x-axis are semicircles. Find the volume of the solid.

(iii) Find the volume of a solid pyramid with square base that is 5 units tall and 20 units on the side.

(iv) A regular cone has a base that is 4 units across and 5 units tall. Find the volume of the cone.

(v) The base of a solid has boundary given by $y = 4 - x^2/9$ and $y = 0$. Cross sections perpendicular to the x-axis are rectangles with heights twice the length of the side lying in the plane. Find the volume of this solid.

(vi) The base of a solid has boundary given by $y = \sqrt{4 - x^2}$ and $y = 0$. Cross sections parallel to the x-axis are squares. Find the volume of the solid.

(vii) The base of a solid has boundary given by the ellipse $4x^2 + 9y^2 = 9$. Cross sections perpendicular to the x-axis are isosceles right triangles with the hypotenuse lying in the plane. Find the volume of the solid.

(viii) The base of a solid has boundary given by $x^2 + y^2 = 4$. The cross sections perpendicular to the x-axis are equilateral triangles. Find the volume of the solid.

(ix) The base of a solid is given by the curve $y = \sin x$ from 0 to π and the curve $y = 0$. Cross sections perpendicular to the x-axis are semicircles. Find the volume of the solid.

(x) The base of a solid is given by the curves $y = \sqrt{x}$ and $y = x^2$. Slices perpendicular to the y-axis are rectangles with height a third the length of the side lying in the plane. Find the volume of the solid.