Problem 1: Division: Integrate the following:

(a) $\displaystyle\int \frac{x^2 - 2x - 15}{x - 5}\,dx$

(b) $\displaystyle\int \frac{2x^2 + 9x + 4}{x + 4}\,dx$

(c) $\displaystyle\int \frac{x^2 - 3x + 1}{x - 3}\,dx$

(d) $\displaystyle\int \frac{2x^2 - 11x + 13}{x - 3}\,dx$

(e) $\displaystyle\int \frac{3x^3 + 3x^2 + 4x + 5}{x + 1}\,dx$

(f) $\displaystyle\int \frac{x^3 - 4x^2 - 3x - 1}{x + 1}\,dx$

(g) $\displaystyle\int \frac{x^3 + 3x^2 + x - 1}{x^2 + 1}\,dx$

(h) $\displaystyle\int \frac{x^4 + 3x^3 + 3x - 4}{x^2 + 1}\,dx$

(i) $\displaystyle\int \frac{x^3 - 3x^2 + 4x - 7}{4 + x^2}\,dx$

(j) $\displaystyle\int \frac{9x^3 + 9x^2 + 4x + 2}{9x^2 + 4}\,dx$

(k) $\displaystyle\int \frac{5x^4 + 33x^2 + 17}{5x^2 + 3}\,dx$

Problem 2: Linear Factors: Integrate the following:

(a) $\displaystyle\int \frac{2x - 1}{x^2 - x - 2}\,dx$

(b) $\displaystyle\int \frac{2x - 24}{x^2 - x - 6}\,dx$

(c) $\displaystyle\int \frac{3x + 5}{x^2 - x - 42}\,dx$

(d) $\displaystyle\int \frac{9 - 4x}{x^2 + 3x}\,dx$

(e) $\displaystyle\int \frac{4x - 2}{x^3 - x}\,dx$

(f) $\displaystyle\int \frac{4x^2 + 2x + 18}{(x + 2)(x + 1)(x - 4)}\,dx$

(g) $\displaystyle\int \frac{2x^2 - 5x + 9}{(x - 1)(x + 5)(x - 2)}\,dx$

(h) $\displaystyle\int \frac{6x^2 + x - 10}{(x + 1)(x + 2)(x - 4)}\,dx$

Problem 3: Linear Powers: Integrate the following:

(a) $\displaystyle\int \frac{x^2 + 3x - 1}{(x - 2)(x + 1)^2}\,dx$

(b) $\displaystyle\int \frac{3x^2 + 14x + 2}{(x - 5)(x + 2)^2}\,dx$

(c) $\displaystyle\int \frac{4 + x - x^2}{(x + 1)^2(x + 2)}\,dx$

(d) $\displaystyle\int \frac{15 - 4x - 4x^2}{3x^2 + x^3}\,dx$

(e) $\displaystyle\int \frac{5x^2 + 28x + 31}{(x + 3)^2(x - 1)}\,dx$

(f) $\displaystyle\int \frac{6x^2 + 16x + 11}{(x + 1)(x + 2)^2}\,dx$

(g) $\displaystyle\int \frac{2x^3 + 11x^2 + 19x + 11}{(x + 1)^2(x + 2)^2}\,dx$

(h) $\displaystyle\int \frac{8x^3 - 15x^2 - 61x + 77}{(x - 3)^2(x + 2)^2}\,dx$

(i) $\displaystyle\int \frac{2x^4 + 2x^3 + 2x^2 + 2x + 1}{x^3 + 2x^4 + x^5}\,dx$

(j) $\displaystyle\int \frac{5x^4 - 11x^3 - 19x^2 + 42x - 9}{(x + 1)^2(x - 2)^3}\,dx$

Problem 4: Simple Quadratic: Integrate the following:

(a) $\int \dfrac{2x+3}{x^2+3x+1}\,dx$ (c) $\int \dfrac{2x-1}{x^2-x+8}\,dx$ (e) $\int \dfrac{6x}{x^2+2}\,dx$

(b) $\int \dfrac{2x}{x^2+9}\,dx$ (d) $\int \dfrac{-4x}{x^2-5}\,dx$ (f) $\int \dfrac{10x}{3x^2+5}\,dx$

Problem 5: Quadratic Alone: Integrate the following:

(a) $\int \dfrac{dx}{x^2+1}\,dx$ (f) $\int \dfrac{dx}{3x^2+2}\,dx$

(b) $\int \dfrac{dx}{x^2+4}\,dx$ (g) $\int \dfrac{x^2+2x+1}{(x^2+1)^2}\,dx$

(c) $\int \dfrac{dx}{4x^2+9}\,dx$ (h) $\int \dfrac{x^2+3x+4}{(x^2+4)^2}\,dx$

(d) $\int \dfrac{dx}{5x^2+4}\,dx$ (i) $\int \dfrac{9x^2-2x+12}{(3x^2+4)^2}\,dx$

(e) $\int \dfrac{dx}{4x^2+5}\,dx$ (j) $\int \dfrac{6x-25-35x^2}{(7x^2+5)^2}\,dx$

Problem 6: Linear/Quadratic: Integrate the following:

(a) $\int \dfrac{2x^2+3x-4}{x^3-2x^2+x-2}\,dx$ (f) $\int \dfrac{8x^2-14x+36}{(x-2)(x^2+4)}\,dx$

(b) $\int \dfrac{7x^2+8x+5}{x^3+x^2+x+1}\,dx$ (g) $\int \dfrac{2x^3+3x^2+16x+21}{(x+5)(x+2)(x^2+1)}\,dx$

(c) $\int \dfrac{2x^2+4x+16}{x^3+2x^2+4x+8}\,dx$ (h) $\int \dfrac{3+4x^2-2x^3}{x^2+x^4}\,dx$

(d) $\int \dfrac{5x^2-10x+4}{(x-5)(3x^2+4)}\,dx$ (i) $\int \dfrac{2x^2-1}{x^2+x^4}\,dx$

(e) $\int \dfrac{3x^3-4x^2+x-8}{(x-3)(x+1)(x^2+1)}\,dx$ (j) $\int \dfrac{7-5x+4x^2-x^3}{(x-2)^2(x^2+1)}\,dx$

Problem 7: Challenge: Integrate the following:

(a) $\int \dfrac{4x^4+12x^3+33x^2+48x-37}{(x+1)^2(x-2)(x^2+9)}\,dx$ (d) $\int \dfrac{x^6+x^5-5x^4-3x^3+10x^2+5x+4}{x(x+2)^2(x^2+1)}\,dx$

(b) $\int \dfrac{x^5-2x^4+4x^3-x^2+x+2}{x^2+x^4}\,dx$ (e) $\int \dfrac{2x^6-8x^5+12x^4-35x^3+10x^2-10x-15}{x^2(x-3)(2x^2+5)}\,dx$

(c) $\int \dfrac{x^4+3x^3+5x^2+10x}{(x+2)(x^2+4)}\,dx$