Math 222: Exam 1
Spring – 2017
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80 Minutes

Name: *Caleb M^c^Whorter — Solutions*

Write your name on the appropriate line on the exam cover sheet. This exam contains 7 pages (including this cover page) and 6 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total:	60	

1. (10 points) Mark the following statements as T for True or F for False. No justification is required.

   (a) ___T___: As the number of observations in an experiment increases, the t–distribution 'looks' more like the normal distribution.

   (b) ___F___: If the null hypothesis is not rejected, there is strong evidence that the null hypothesis is true.

   (c) ___F___: Hypothesis testing can only be used when the mean of the population is known.

   (d) ___T___: All other things equal, choosing a smaller significance level, α, will increase the probability of making a Type II error.

   (e) ___T___: The core principle of hypothesis testing is to reject H_0 only when the observed sample is unlikely to have occurred when H_0 is true.

   (f) ___F___: Inferences about population proportions are insensitive to false replies in the data.

   (g) ___F___: A matched paired t procedure can only be used when the same individuals are tested in each group.

   (h) ___T___: The greater the difference in the two sample sizes, the more sensitive a pooled t procedure will be to violations of the required assumptions.

   (i) ___F___: The power of a test, β, is the probability of failing to reject a false null hypothesis.

   (j) ___T___: The sampling distribution of n individuals from a normally distributed population is always normally distributed.

2. (10 points) You are looking to replace your older vehicle which only gets 27 miles per gallon (mpg) with a newer model that gets at least 32 mpg. After doing some research, you find a good match for a possible new car; SNW, a small four–door car company, claims their newest 2017 model car averages 32 miles per gallon. However after reading a few online reviews quoting varying mpg values, you begin to question this claim. You find quoted mpg of 33, 27, 32, 28, 32, 26, 29, and 30. After some calculations, you find this data set is normal with mean of 29.625 and standard deviation 2.560.

   (a) Construct a 95% confidence interval for the mpg for SNW's latest vehicle.

   The population standard deviation is unknown, so we use a one-sample t-procedure. Though we have $n = 8$, this is appropriate because the sample is known to be normal. We have $\overline{x} = 29.625$, $s = 2.560$, and $n = 8$ (which means we have degrees of freedom 7). Using a 95% confidence level and degrees of freedom 7, we know that $t^* = 2.365$. But then

$$\begin{aligned} \overline{x} &\pm t^* \frac{s}{\sqrt{n}} \\ 29.625 &\pm 2.365 \frac{2.560}{\sqrt{8}} \\ 29.625 &\pm 2.14055 \end{aligned}$$

   This gives confidence interval $(27.4845, 31.7655)$. Therefore, we are 95% certain that the true average mpg for this car is between 27.5 mpg and 31.8 mpg.

   (b) Is the company's claim that the mpg of the vehicle being exactly 32 mpg believable [at the 5% significance level]?

   Notice that 32 mpg is not in the 95% confidence interval, so we do not expect the company's claim to be believable. We choose null and alternative hypotheses

$$\begin{cases} H_0 : \mu = 32 \\ H_a : \mu \neq 32 \end{cases}$$

$$t = \frac{\overline{x} - \mu}{s/\sqrt{n}} = \frac{29.625 - 32}{2.560/\sqrt{8}} = -2.624 \rightsquigarrow 0.02$$

   Then we have $p = 2 \cdot 0.02 = 0.04$. At the 5% significance level, there is sufficient evidence to suggest that this car does not get an average of 32 mpg.

3. (10 points) A drug company produces a new hormone treatment to help those with particular mental illnesses. Due to differences in hormone levels in males and females, the company wants to see if the drug is more/less effective between the genders. Out of the estimated 237,345 individuals in the United States with the applicable mental illnesses, the company tests the treatment on 97 women and 112 men. The treatment was found to be effective for 71 of the females and 84 of the males.

   (a) Construct a 90% confidence interval for the difference of effectiveness of the treatment between the genders.

   The population proportions are unknown so a two-sample proportion inference is used. We know the procedure is valid as there are at least 10 success/failures in each group. We have $x_F = 71$, $n_F = 97$, $\hat{p}_F = 71/97 = 0.732$ *and* $x_M = 84$, $n_M = 112$, *and* $\hat{p} = 84/112 = 0.75$. *Then*

$$\hat{p}_F - \hat{p}_M = -0.0180412$$
$$\sigma_D = \sqrt{\frac{0.732(1-0.732)}{97} + \frac{0.75(1-0.75)}{112}} = 0.0608$$

   Using a 90% confidence level, we have $z^* = 1.645$, *which gives*

$$\begin{array}{rcl} (\hat{p}_F - \hat{p}_M) & \pm & z^*\sigma_D \\ -0.0180412 & \pm & 1.645(0.0608) \\ -0.0180412 & \pm & 0.10000 \end{array}$$

   This gives confidence interval $(-0.118, 0.0819)$. *Therefore, we are 90% certain that the treatment is on average 11.8% less effective to 8.19% more effective for females than for males.*

   (b) Using a significance level of 1%, is there evidence that the treatment is more/less effective for one gender?

   We use hypotheses

$$\begin{cases} H_0 : \hat{p}_F = \hat{p}_M \\ H_a : \hat{p}_F \neq \hat{p}_M \end{cases}$$

   We have $\hat{p} = \dfrac{71+84}{97+112} = 0.7416$ *and* $\sigma_p = \sqrt{0.7416(1-0.7416)\left(\dfrac{1}{97} + \dfrac{1}{112}\right)} = 0.0607$. *Then*

$$z = \frac{0.732 - 0.75}{0.0607} = -0.30 \rightsquigarrow 0.3821$$

   Then $p = 2 \cdot 0.3821 = 0.7642$. *Using a significance level of 1%, there is not sufficient evidence to suggest there is a difference in treatment effectiveness between the genders. Note that we could have seen this from the confidence interval as* $0 \in (-0.118, 0.0819)$.

4. (10 points) A university gives its students a college Mathematics preparedness exam to determine if the new incoming students are ready for their college mathematics classes. The instructor notices that their female students tend to be able to manage their mathematics classes better than the typical male student. The professor wonders if this could have been predicted using the preparedness exam scores. Examining 10 incoming female freshmen scores, the professor records a mean of 91.9 with standard deviation 2.7 while for 11 incoming male freshmen students the professor finds a mean of 87.8 with standard deviation 5.1. Both samples were normally distributed.

   (a) Using this data, construct a 90% confidence interval for the difference in female–male incoming freshman exam performance.

   The population standard deviations are unknown so a two-sample t*-procedure is used. Though the sample sizes are at most size 11, this is appropriate as the distributions are normal. We have* $\overline{x}_F = 91.9$*,* $s_F = 2.7$*,* $n_F = 10$ *and* $\overline{x}_M = 87.8$*,* $s_M = 5.1$*, and* $n_M = 11$*. We have degrees of freedom* $\min\{10-1, 11-1\} = 9$*, which gives for a 90% confidence interval* $t* = 1.833$*. Then*

$$\begin{aligned}(\overline{x}_F - \overline{x}_M) &\pm t^*\sqrt{\frac{s_F^2}{n_F} + \frac{s_M^2}{n_M}} \\ 4.1 &\pm 1.833\sqrt{\frac{2.7^2}{10} + \frac{5.1^2}{11}} \\ 4.1 &\pm 1.833(1.7589) \\ 4.1 &\pm 3.2241\end{aligned}$$

   This gives 90% confidence interval $(0.876, 7.324)$*. Therefore, we are 90% certain that females score on average 0.876 to 7.324 percent better than males on the preparedness exam.*

   (b) Using a significance level of 5%, is there sufficient evidence that a typical female student scores higher than their male counterpart?

   We use null and alternative hypotheses

$$\begin{cases} H_0 : \mu_F = \mu_M \\ \\ H_a : \mu_F > \mu_M \end{cases}$$

$$t = \frac{\overline{x}_F - \overline{x}_M}{\sqrt{\frac{s_F^2}{n_F} + \frac{s_M^2}{n_M}}} = \frac{4.1}{1.7589} = 2.331 \rightsquigarrow 0.025$$

   We know that the true probability p *is* $0.02 < p < 0.025$*. In either case, there is sufficient evidence to suggest that on average females perform better than males on average on the Mathematics preparedness exam.*

5. (10 points) A common aliment among the elderly is arthritis—a painful, chronic inflammation of the joints in the body. This inflammation is typically located in the extremities, such as the hands, but can also be found in joints such as the hip. Some doctors suggest taking a regular anti-inflammatory medication, such as ibuprofen, to help alleviate the symptoms. However, this medication can have adverse effects on the liver if taken in too large of a dose. Researchers test 392 nursing home patients and find that 18 of the patients taking the medication suffer these effects.

   (a) Construct a 95% confidence interval for the average amount of patients that will suffer these side–effects. The company producing the drug gives data suggesting 3% or less of patients will suffer from these side effects.

   The population proportion is unknown so we use a one-sample proportion inference. This is appropriate as there are at least 10 successes/failures and the sample size is small relative to the population. We have $X = 18$*,* $n = 392$*, and* $\hat{p} = 18/392$*. Using a 95% confidence interval, we have* $z^* = 1.960$*. Then we have*

$$\begin{array}{rcl} \hat{p} & \pm & z^* \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}} \\ 0.0459 & \pm & 1.960\sqrt{\dfrac{0.0459(1-0.0459)}{392}} \\ 0.0459 & \pm & 1.960(0.0105717) \\ 0.0459 & \pm & 0.0207 \end{array}$$

   This gives 95% confidence interval $(0.0252, 0.0666)$*. Therefore, we are 95% certain that between 2.52% and 6.66% of patients will suffer adverse side effects.*

   (b) Using a 5% confidence level, is there evidence suggesting that the elderly suffer the side effects more than typical ibuprofen user?

   We have $p_0 = 0.03$*. We use null and alternative hypotheses*

$$\begin{cases} H_0 : p_0 = 0.03 \\ \\ H_a : p_0 > 0.03 \end{cases}$$

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)n}} = \frac{0.0459 - 0.03}{\sqrt{\dfrac{0.03(1-0.03)}{392}}} = \frac{0.0159}{0.008616} = 1.845 \approx 1.85 \rightsquigarrow 0.9678$$

   Therefore, $p = 1 - 0.9678 = 0.0322$*. Therefore at the 5% significance level, there is sufficient evidence to suggest that elderly regular ibuprofen users experience side effects more often on average than the typical ibuprofen user.*

6. (10 points) A educational methodology researcher wants to compare interactive computer-based learning with interactive hands-on learning in 6^{th} grade Chemistry. The researcher contacts a local school to gather participants for the study. The school and parents agree and 12 sixth grade students are selected. The students of equal ability are paired into groups, one given computer instruction and the other hands-on instruction. At the end of the week, the students are given an exam on the topics covered. The results are summarized below:

Group	Computer	Hands–On
1	73	72
2	71	75
3	77	76
4	73	77
5	77	78
6	75	77

Assuming grades for any interactive learning method are typically normally distributed, test whether there is a difference in the two teaching methods at a 10% significance level.

The population standard deviations are unknown and the samples are not independent. Therefore, we use the matched pair t-procedures. The sample size is small but we know that the samples are normally distributed. We have differences

Group	*1*	*2*	*3*	*4*	*5*	*6*
Difference	-1	4	-1	4	1	2

We have $\overline{x}_D = 1.50$ and $s_D = 2.25832$. Furthermore, $n = 6$ so we have degrees of freedom $n - 1 = 5$. We use null and alternative hypotheses

$$\begin{cases} H_0 : \mu_D = 0 \\ \\ H_a : \mu_D \neq 0 \end{cases}$$

$$t = \frac{\overline{x}_D}{s_D/\sqrt{n}} = \frac{1.50}{2.258/\sqrt{6}} = \frac{1.50}{0.921825} = 1.627 \rightsquigarrow 0.10$$

Therefore, $p = 2 \cdot 10 = 0.20$. Therefore at the 10% significance level, there is not sufficient evidence to suggest that there is any difference in student learning (based on this exam) between the two teaching methods.