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Quiz 1 **Spring 2017**

Problem 1: A machine is used to fill soda bottles. The amount of soda dispensed into each bottle varies slightly and is known to have a normal distribution with population standard deviation 2.62 mL. A random sample of 25 bottles filled by the machine is taken and the amount of soda filled in each bottle was measured. From this sample data, the sample mean was calculated to be 591.2 mL. Find a 94% confidence interval for the population mean amount of soda filled by the machine. State this conclusion in words.

We know that $\overline{x} = 591.2$ *mL,* $\sigma = 2.62$ *mL, and* $n = 25$*. Because the population standard deviation is known and the underlying distribution is normal, a* z*-approach is appropriate. To construct a 94% confidence interval, we need the* z^* *corresponding to 97%. This gives* $z^* \mapsto 0.97$ *so that* $z^* = 1.88$*. We know that*

$$\begin{aligned} \overline{x} &\pm z^* \frac{\sigma}{\sqrt{n}} \\ 591.2 &\pm (1.88)(0.524) \\ 591.2 &\pm 0.98512 \end{aligned}$$

so that we have 94% confidence interval $(590.21, 592.19)$*. Therefore, we are 94% confident that the mean amount of soda dispensed into each bottle is between 590.21 mL and 592.19 mL.*

Problem 2: The level of calcium in the blood of healthy young adults follows a normal distribution with mean $\mu = 10$ milligrams per deciliter and standard deviation $\sigma = 0.5$. A clinic measures the blood calcium of 27 healthy pregnant young women at their first visit for prenatal care. The mean of these 27 measurements is $\overline{x} = 9.8$. Is this evidence that the mean calcium level in the population of healthy pregnant young women is less than 10? To determine this, follow the following steps:

(a) Set up the null and alternative hypotheses that that you will be testing in the above scenario.

$$\begin{cases} H_0 : \mu = 10 \\ \\ H_a : \mu < 10 \end{cases}$$

(b) Assume that the distribution of calcium level measurements for pregnant young women is a normal distribution with $\sigma = 0.5$, find the value of the test statistics and compute the p–value.

We have $\overline{x} = 9.8$, $\sigma = 0.5$, and $n = 27$. A z-procedure is appropriate as the underlying distribution is normal. Then

$$z_{9.8} = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} = \frac{9.8 - 10}{0.5/\sqrt{27}} = \frac{-0.2}{0.096} = -2.08 \rightsquigarrow 0.0188$$

Therefore, $p = 0.0188$.

(c) Is this statistically significant at the 5% level? Is this statistically significant at the 1% level?

If $\alpha = 0.05$, then $p < \alpha$ so that the result is significant. If $\alpha = 0.01$, then $p > \alpha$ so that the result is not significant.

(d) State your conclusion in words at the 5% significance level.

Because $p = 0.0188 < \alpha = 0.05$, the result is significant. Therefore, we reject the null hypothesis. There is sufficient evidence to believe that the average calcium level of pregnant young women is not 10 mg in favor of the alternative that, on average, it is less than 10 mg on average.