Name: *Caleb McWhorter — Solutions* **MAT 222**
Quiz 3 **Spring 2017**

Problem 1: Mark the following statements as true or false in the blank space provided. If the statement is false, explain why it is false.

(a) ___F___: If the null hypothesis is not rejected, there is strong evidence that the null hypothesis is true.

Failing to reject a hypothesis does not make it true or false.

(b) ___T___: All other things equal, choosing a smaller significance level, α, will increase the probability of making a Type II error.

One can draw a diagram for this. Intuitively, decreasing the significance level makes it more difficult to reject any hypothesis. But then it makes it more difficult to reject false null hypotheses, which increases the probability of a Type II error.

(c) ___T___: The core principle of hypothesis testing is to reject H_0 only when the observed sample is unlikely to have occurred when H_0 is true.

This is exactly hypothesis testing: any event could occur by chance, but if the chance is too 'small', then it is more likely that the mean is actually something else so that this event is not as unusual as it seems.

(d) ___F___: The power of a test, β, is the probability of failing to reject the null hypothesis.

Power, β, is the probability of rejecting a false null hypothesis. Therefore, the probability of failing to reject a false null hypothesis is $1 - \beta$. The probability of failing to reject a (true) null hypothesis is $1 - \alpha$.

(e) ___T___: As the number of observations in an experiment increases, the t–distribution 'looks' more like the normal distribution.

The larger the n, the more the Central Limit Theorem and the closer the sample standard deviation approaches the true population standard deviation. Therefore, the t-distribution approaches the normal distribution.

Problem 2: A faster loan processing time produces higher productivity and greater customer satisfaction. A financial services institution wants to determine if their mean loan processing time is less than a competitor's claim of 6 hours. A financial analyst randomly selects 7 loan applications and manually calculates the time between loan initiation and when the customer receives the institution's decision. From the sample data, the sample mean of the loan processing time was 5.079 hours with a sample standard deviation of 1.319 hours. Assuming that the loan processing times follow a normal distribution, complete the following parts. You must show all necessary calculations and provide explanation.

(a) Find a 95% confidence interval for the population mean loan processing time.

We know that $\overline{x} = 5.079$, $s = 1.319$, *and* $n = 7$. *Because the population standard deviation is unknown and the underlying population distribution is normal, a* t*-procedure is appropriate. We have degrees of freedom* $n - 1 = 6$, *so that for a 95% confidence level we have* $t^* = 2.447$. *Then*

$$\begin{array}{ccl} \overline{x} & \pm & t^* \dfrac{s}{\sqrt{n}} \\ 5.079 & \pm & 2.447 \dfrac{1.319}{\sqrt{7}} \\ 5.079 & \pm & 2.447 \cdot 0.4985 \\ 5.079 & \pm & 1.220 \end{array}$$

which gives 95% *confidence interval* $(3.86, 6.30)$. *Therefore, we are 95% certain that this loan company takes between 3.86 and 6.30 hours to processes a loan, on average.*

(b) Compute the test statistic and the p–value that can be used to determine if their mean time is less than a competitor's claim of 6 hours.

$$\begin{cases} H_0 : \mu = 6 \\ \\ H_a : \mu < 6 \end{cases}$$

$$t = \frac{\overline{x} - \mu}{s/\sqrt{n}} = \frac{5.079 - 6}{0.499} = \frac{-0.921}{0.499} = -1.846 \rightsquigarrow 0.05 < p < 0.10$$

We have test statistic $t = -1.846$ *and* p*-value* $0.05 < p < 0.10$.

(c) State the decisions at significance levels $\alpha = 10\%, 5\%,$ and 1%.

For $\alpha = 0.10$ *(because* $p < \alpha$*), there is sufficient evidence to suggest the company's average loan processing time is less than 6 hours. However for* $\alpha = 0.05$ *and* $\alpha = 0.01$*, there is not sufficient evidence to suggest that the mean loan processing time is less than 6 hours.*