Math 397: Exam 1 **Name:** _____
Summer Session II – 2017
07/17/2017
90 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 11 pages (including this cover page) and 9 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

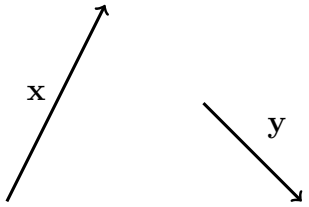
Question	Points	Score
1	9	
2	8	
3	7	
4	6	
5	9	
6	7	
7	7	
8	10	
9	9	
Total:	72	

1. (9 points) Answer the following questions:

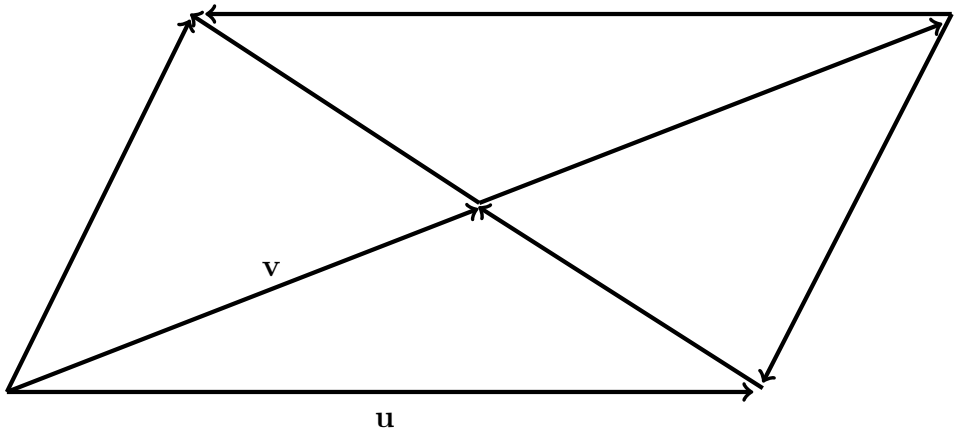
   (a) If $\mathbf{a} = \langle 2, -1, 0 \rangle$ and $\mathbf{b} = \langle 5, -3, 4 \rangle$, find $|\mathbf{a}|$ and $2\mathbf{a} - \mathbf{b}$.

   (b) Find a unit vector in the direction of the vector $\langle 3, 1 \rangle$.

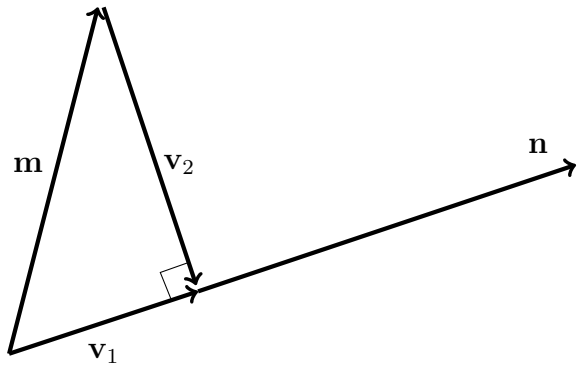
   (c) Given the vectors $\mathbf{x}$ and $\mathbf{y}$ shown below, find $\mathbf{x} + \mathbf{y}$ and $\mathbf{y} - \mathbf{x}$.



(d) In the vector diagram below, label all vectors in terms of the given vectors $\mathbf{u}$ and $\mathbf{v}$.



(e) Given the vector diagram below, find $\mathbf{v}_1$ and $\mathbf{v}_2$ in terms of $\mathbf{m}$ and $\mathbf{n}$.



2. (8 points) Complete the following parts:

   (a) Let $\mathbf{u} = 2\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{k}}$ and $\mathbf{v} = 5\,\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\,\hat{\mathbf{k}}$, find $\mathbf{u} \cdot \mathbf{v}$.

   (b) Find a nonzero vector perpendicular to $5\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}} - \hat{\mathbf{k}}$.

   (c) The curves $y = x^2 - 1$ and $y = \dfrac{5}{2}x - 2$ intersect at the point $(2, 3)$, find the acute angle between the curves at this point of intersection.

3. (7 points) Complete the following parts:

   (a) Find a vector perpendicular to both $\mathbf{u} = \langle 0, -1, 1 \rangle$ and $\mathbf{v} = \langle 5, -3, -1 \rangle$.

   (b) Find the area of the triangle with vertices $P(1, 0, 1)$, $Q(-1, 1, 1)$, and $R(0, 2, 1)$.

   (c) Find the volume of the parallelepiped formed by $\mathbf{u} = \langle 2, 1, -1 \rangle$, $\mathbf{v} = \langle 1, 2, 4 \rangle$, and $\mathbf{w} = \langle 2, 2, 2 \rangle$. What, if anything, does this answer imply about $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$?

4. (6 points) Find a parametrization for the following curves:

   (a) The line segment from the point $(7, -1, 4)$ to the point $(-3, 2, 3)$.

   (b) The ellipse, oriented counterclockwise, given by

$$\frac{(x-2)^2}{3} + \frac{(y+1)^2}{5} = 1$$

   (c) The curve given by $f(x) = x\cos x + e^x \sin^2 \sqrt[3]{x}$.

5. (9 points) Suppose the path of a photon emitted from a hydrogen atom is given by $\mathbf{x}(t) = (t, \cos t, \sin t)$.

   (a) Sketch the path of the particle. Find the position of the particle at $t = \pi$.

   (b) Find the velocity of the particle at $t = \dfrac{\pi}{4}$.

   (c) If the electron orbiting the atom has acceleration vector given by $\mathbf{a}(t) = 2\,\hat{\mathbf{i}} - \cos(1-t)\,\hat{\mathbf{k}}$ and the electron was located at $2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$ with velocity vector $2\hat{\mathbf{i}}$ at $t = 1$, give a vector valued function for the electron's position.

6. (7 points) Complete the following parts:

   (a) Find the (vector) parametric equation of the line containing the point $(0, 5, 3)$ and parallel to the vector $\langle 2, -3, 1\rangle$.

   (b) Find the (vector) parametric equation of the line perpendicular to the plane $x - 3y = 13$ and containing the point $(-1, 0, 1)$.

   (c) Determine if the lines $l_1(t) = (t, 1 - t, 1 - t)$ and $l_2(s) = (6s, -s - 1, 3s + 1)$ are the same, parallel, skew, or intersecting. If the lines intersect, find the point of intersection and determine if they are perpendicular.

7. (7 points) Complete the following parts:

   (a) Find a normal vector to the plane $5x + 3y + z = 15$ and find three points on the plane.

   (b) Find the equation of the plane with normal vector $\langle 2, 3, -1 \rangle$ and containing the point $(1, 0, 2)$.

   (c) Find the equation of the plane parallel to the plane $2x - y = 5 - 3z$ and containing the line $l(t) = (-1, 10, 4)t + (2, 5, 2)$.

8. (10 points) Complete the following parts:

   (a) Find the distance between the planes $2x - 5y + z = 4$ and $2x - 5y + z = 6$.

   (b) Find a parametrization for the intersection of $\Pi_1 : x+y-z = 4$ and $\Pi_2 : x-y+z = 6$.

9. (9 points)

(a) Identify the surfaces listed below:

(i) $x + y = 2z - 4$ _____

(ii) $(x-1)^2 + y^2 + (z+2)^2 = 5$ _____

(iii) $z = 1 - 2x^2 - 3y^2$ _____

(iv) $y^2 - z^2 = x^2 + 1$ _____

(v) $x = y^2$ _____

(vi) $y = \dfrac{z^2}{4} - x^2$ _____

(vii) $y^2 + z^2 = 4$ _____

(viii) $x^2 + y^2 - z^2 = 4$ _____

(b) Use appropriate level curves to sketch the surface $y^2 = x^2 + \dfrac{z^2}{4}$.