Math 397: Exam 2 **Name:** ______________________________
Summer Session II – 2017
07/27/2017
120 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 13 pages (including this cover page) and 8 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	12	
2	10	
3	16	
4	10	
5	10	
6	15	
7	15	
8	12	
Total:	100	

1. (12 points) Evaluate the following limits. If the limit exists, prove that it exists; if the limit does not exist, explain why.

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 - xy + y^2}{x^2 + y^2}$

   (b) $\displaystyle\lim_{(x,y)\to(1,0)} \frac{6xe^y - \sin y \cos x}{x^2 - y^2}$

(c) $\displaystyle\lim_{(x,y)\to(2,1)} \frac{x^4 - 2x^3y - x^2y^2 + 2xy^3}{x^2 - 2xy}$

(d) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 \sin^2 y}{3x^2 + 2y^2}$

2. (10 points) Let $f(x, y, z) := \dfrac{x^y + y\ln(2y) - z}{\cos x}$. Find the following partial derivatives [you need not simplify]:

(a) $\dfrac{\partial f}{\partial x} =$

(b) $\dfrac{\partial f}{\partial y} =$

(c) $\dfrac{\partial f}{\partial z} =$

(d) $\dfrac{\partial^2 f}{\partial y \partial x} =$

(e) $\dfrac{\partial^2 f}{\partial z^2} =$

3. (16 points) Complete the following parts:

(a) Set up *but do not evaluate* an integral find the arclength of the curve given by $x(t) = \sqrt{2}\,t$, $y(t) = \dfrac{t^2}{2}$, and $z(t) = \ln t$ for $1 \leq t \leq 4$.

(b) Find the tangent plane to the function $f(x, y, z) = x^2y + 3z\cos(yz)$ at the point $(1, 2, 0)$.

(c) Find the total differential for the function $f(x, y, z) = \dfrac{y\sqrt[3]{x}}{z^2}$.

(d) Use the method of total differentials to approximate $\sqrt[3]{7.4} \cdot \dfrac{0.9}{(1.1)^2}$.

4. (10 points) Complete the following parts:

(a) Let $f(x,y,z) = x^2z + yz^2 - \cos(xyz)$. Find $\nabla f(1,\pi,1)$.

Let $\mathbf{V}(x,y,z) = (x+yz)\,\hat{\mathbf{i}} + (y+xz)\,\hat{\mathbf{j}} + (z+xy)\,\hat{\mathbf{k}}$.

(b) Find $\mathrm{Div}\,\mathbf{V}$.

(c) Find $\mathrm{curl}\,\mathbf{V}$.

5. (10 points) Let S be the surface given by $f(x, y) = x^2 - \dfrac{1}{y^2}$.

(a) Find $D_{\mathbf{u}}f(2, -1)$, where $\mathbf{u} = \langle 1, -1 \rangle$.

(b) Find the direction of steepest ascent and descent on this surface at $(2, -1)$.

(c) Find the maximum and minimum rates of change for $f(x, y)$ at the point $(2, -1)$.

6. (15 points) Identify and determine the nature of any critical points for the function $f(x, y, z) := x^3 + xz^2 - 3x^2 + y^2 + 2z^2$.

7. (15 points) Find the maximum volume of a rectangular box contained in the ellipsoid $x^2 + 9y^2 + 4z^2 = 9$. [Hint: Explain why this box must have vertices lying on the ellipsoid and why one of these 'corners' must lie in the first octant. Call this point (x, y, z). Express the volume of the box in terms of this point and proceed.]

8. (12 points) Complete the following parts:

(a) Sketch the region of integration and evaluate $\int_0^2 \int_{-e^x}^{e^x} 3y^2 \, dy \, dx$.

(b) Set up *but do not evaluate* the bounds for the integral $\iint_R xy \, dA$, where R is the region enclosed by $y = \dfrac{x}{2}$, $y = \sqrt{x}$, $x = 2$, and $x = 4$.

(c) Evaluate $\displaystyle\int_0^4 \int_{\sqrt{x}}^2 \frac{10x}{1+y^5}\, dy\, dx$.