Geometry Proofs

In High School, one has to painstakingly work through a variety of geometric proofs using coordinates. The purpose of this project will be to work through a few familiar theorems from High School, as well as a few important new ones, using simple vector arguments.

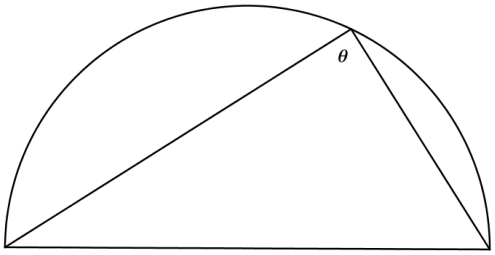
1 Parallelogram Law

Let $\mathbf{a}$ and $\mathbf{b}$ be arbitrary vectors. Prove the Parallelogram Law:

$$|\mathbf{a}+\mathbf{b}|^2 + |\mathbf{a}-\mathbf{b}|^2 = 2\left(\,|\mathbf{a}|^2 + |\mathbf{b}|^2\,\right)$$

Give a geometric interpretation to this equality.

2 Thales' Theorem



Show that any angle inscribed in a circle whose endpoints lie along a diagonal of the circle is a right angle.

3 Cauchy–Schwartz and Triangle Inequality

For vectors $\mathbf{a}$ and $\mathbf{b}$ in $\mathbb{R}^3$, prove the Cauchy–Schwartz Inequality:

$$|\mathbf{a}\cdot\mathbf{b}| \leq |\mathbf{a}|\,|\mathbf{b}|$$

Use this to prove the Triangle Inequality:

$$|\mathbf{a}+\mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$$

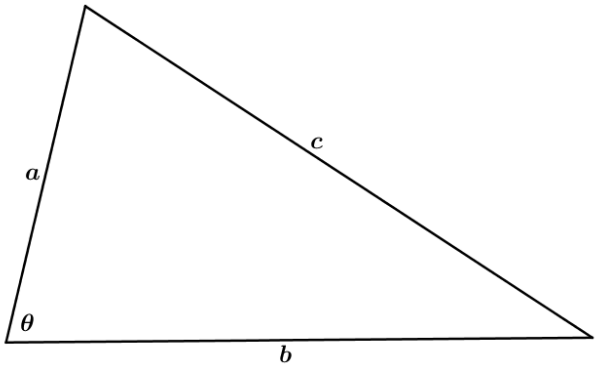
Give a geometric interpretation for the Triangle Inequality.

4 Triangle Area

If a triangle is formed by vectors $\mathbf{a}$ and $\mathbf{b}$, show that the area of the resulting triangle is given by

$$A = \frac{1}{2}\sqrt{|\mathbf{a}|^2\,|\mathbf{b}|^2 - (\mathbf{a}\cdot\mathbf{b})^2}$$

5 Law of Cosines



Prove the Law of Cosines:

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

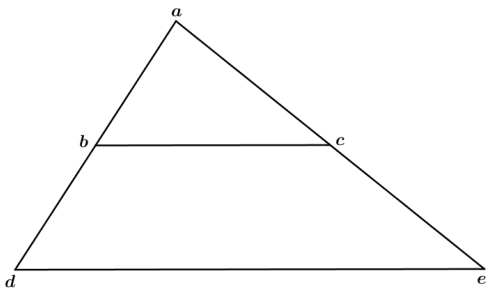
6 Square Proof

Show that if the diagonals of a parallelogram are perpendicular, then the parallelogram is a square.

7 Parallelogram Proof

If the opposite sides of a quadrilateral are parallel and of equal length, then the quadrilateral is a parallelogram.

8 Parallel Proof



Consider a triangle Δadc. Show that connecting the midpoints of $\overline{ad}$ and $\overline{ae}$, labeled b, c, respectively, yields a side parallel to $\overline{de}$ and is half the length of $\overline{de}$.

9 Regular Polygon

Let $P_1, P_2, \ldots, P_n$ denote the vertices of a regular polygon with n–sides and O denote the center of the polygon. Show that $\sum_{i=1}^{n} \vec{OP_i} = \mathbf{0}$.

Evaluation

Complete the following survey by rating each problem. Each area will be rated on a scale of 1 to 5. For interest, 1 is “mind-numbing" while a 5 is “mind-blowing". For difficulty, 1 is “trivial/routine" while 5 is “brutal." For learning, 1 means “nothing new" while 5 means "profound awakening". Then you to estimate the amount of time you spent on each problem (in minutes).

	Interest	Difficulty	Learning	Time Spent
Parallelogram Law				
Thales’ Theorem				
Cauchy–Schwartz & Triangle Inequality				
Triangle Area				
Law of Cosines				
Square Proof				
Parallelogram Proof				
Parallel Proof				
Regular Polygon				

Indicate whether you believe this project was helpful in mastering the course material and/or if it was helpful in developing a deeper understanding of the subject. Also, indicate whether you think this project should be given to future Calculus III students.

	Yes	No
Helpful for the Course		
Helpful in Learning the Subject		
Assign Again		

Finally, you may write any comments, thoughts, or suggestions in the space below.