Math 295: Exam 3 **Name:** ______________________________

Fall – 2018

11/16/2018

50 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 10 pages (including this cover page) and 6 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	15	
2	15	
3	15	
4	20	
5	15	
6	20	
Total:	100	

1. (15 points) Compute the following limits. Be sure to to show all your work and justify your answer completely.

(a) $\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$

(b) $\lim_{x \to \infty} \frac{\ln(1 + e^{6x})}{5x}$

(c) $\lim_{x \to 0^+} \sqrt[3]{x} \ln x$

(d) $\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{xe^{4x}}\right)$

(e) $\lim_{x\to 0}(1+2x)^{3/x}$

2. (15 points) l'Hôpital's Rule is a useful tool but can fail to compute a limit $\lim_{x\to\infty} \frac{f(x)}{g(x)}$ for the following reasons:

   A. $\lim_{x\to\infty} \frac{f'(x)}{g'(x)}$ does not exist.

   B. $\lim_{x\to\infty} \frac{f(x)}{g(x)}$ is not an indeterminate form.

   C. The problem 'loops', i.e. $\lim_{x\to\infty} \frac{f'(x)}{g'(x)}$ is essentially the same limit as the initial limit.

   D. $f(x)$ or $g(x)$ are not differentiable.

   For each of the following limits, indicate one of the reasons above why l'Hôpital's Rule does not apply to the limit.

   ______ $\lim_{x\to\infty} \frac{3x+5}{\sqrt{2x^2+1}}$

   ______ $\lim_{x\to\infty} \frac{x\sin x}{x^2 + x\cos x}$

   ______ $\lim_{x\to\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}}$

   ______ $\lim_{x\to\infty} \frac{2x}{3x+\sin x}$

   Choose one of the limits above and compute it below. Be sure to justify your answer completely.

3. (15 points) Complete each of the following parts. Be sure to show all your work, and justify your answers completely.

   (a) Verify that $f(x) = x^3 + x - 1$ satisfies the hypotheses of the Mean Value Theorem on $[0, 3]$. Find all numbers c satisfying the conclusions of the Mean Value Theorem on this interval.

   (b) Use the Mean Value Theorem to prove that if $f'(x) = 0$ for all $x \in [a, b]$, then $f(x)$ is constant on $[a, b]$. [Hint: Show $f(x_0) = f(a)$ for all $a \leq x_0 \leq b$.]

4. (20 points) A function $f(x)$ and its derivatives $f'(x)$ and $f''(x)$ are given below:

$$f(x) = \left(x - \frac{7}{4}\right) x^{4/3} \qquad f'(x) = \frac{7}{3}x^{1/3}(x-1) \qquad f''(x) = \frac{7(4x-1)}{9x^{2/3}}$$

Answer the following questions. You may show your work on the next page. Be sure to indicate the part.

(a) What are the intervals on which $f(x)$ is increasing?

(b) What are the intervals on which $f(x)$ is decreasing?

(c) Find all critical values for $f(x)$. Classify these critical values as local maxima, local minima, or neither. Be sure to use the First or Second Derivative Test to justify your answer.

(d) What are the intervals on which $f(x)$ is concave?

(e) What are the intervals on which $f(x)$ is convex?

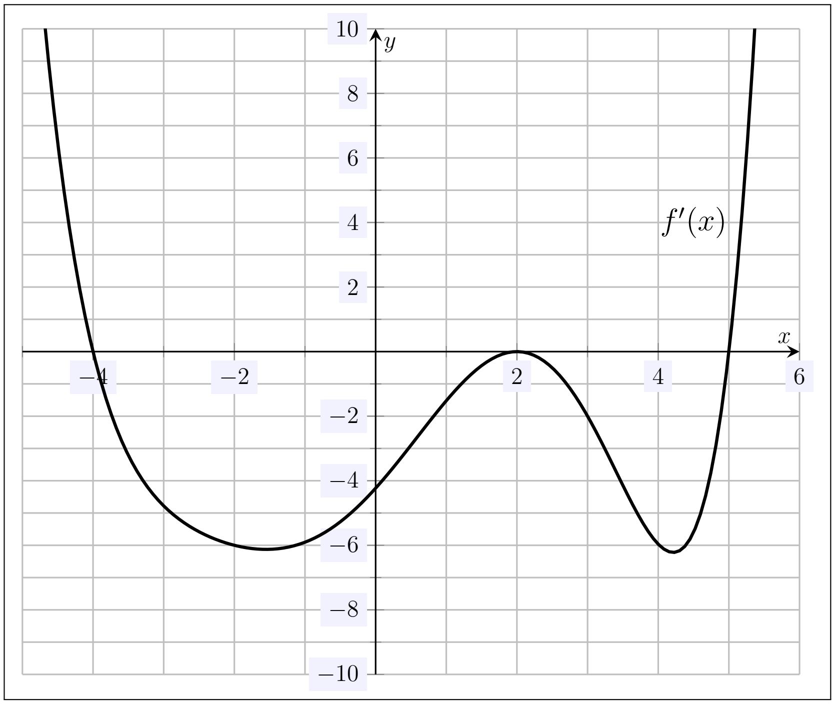
(f) Find the x-values of any points of inflection on $f(x)$.

(g) Find the absolute minimum and absolute maximum *values* of $f(x)$ on $[-1, \frac{7}{4}]$.

$$f(x) = \left(x - \frac{7}{4}\right) x^{4/3} \qquad f'(x) = \frac{7}{3}x^{1/3}(x-1) \qquad f''(x) = \frac{7(4x-1)}{9x^{2/3}}$$

5. (15 points) A rectangular box has a square bottom and an open top. If only 2,700 cm^2 of material is available to construct the box, what dimensions maximize the volume of the box? Be sure to draw a picture and justify completely that these dimensions are optimal.

6. (20 points) The graph of $f'(x)$ for some function $f(x)$ is plotted in the figure below. Based on this graph, complete the questions below.



(a) Find all the intervals on which $f(x)$ is increasing.

(b) Find all the intervals on which $f(x)$ is decreasing.

(c) Find all critical points for $f(x)$. Using the First or Second Derivative Test, classify these x-values as locations of maximums, minimums, or neither.

(d) Find the intervals on which $f(x)$ is concave.

(e) Find the intervals on which $f(x)$ is convex.

(f) Find the x-values of any points of inflection on $f(x)$.