

**l'Hôpital's Rule:**

- Know the indeterminate forms:  $\frac{0}{0}$ ,  $\pm\frac{\infty}{\infty}$ ,  $0 \cdot \infty$ ,  $\infty - \infty$ ,  $1^\infty$ ,  $0^0$ ,  $\infty^0$
- Be able to find limits using l'Hôpital's Rule, especially when more than one application of l'Hôpital's Rule is needed.
- Know how to compute limits involving each of the indeterminate forms above:
  - (i)  $\frac{0}{0}$ ,  $\pm\frac{\infty}{\infty}$ : Handled 'normally.'
  - (ii)  $0 \cdot \infty$ : Move either the 0 or  $\infty$  term into the denominator to obtain one of the above forms.
  - (iii)  $\infty - \infty$ : Combine terms or factor out something to obtain one of the forms above.
  - (iv)  $0 \cdot \infty$ ,  $\infty - \infty$ ,  $1^\infty$ ,  $0^0$ ,  $\infty^0$ : Use logarithms, i.e. set  $L = \lim f(x)^{g(x)}$ , take logs to obtain  $\lim g(x) \ln f(x)$ . Then compute this limit, which is now one of the above indeterminate forms above. If this limit is  $W$ , then the original limit is  $e^W$ .
- Know how to find limits involving indeterminate forms without l'Hôpital's Rule, i.e. multiply by  $\frac{1/x^{\text{deg den}}}{1/x^{\text{deg den}}}$ . This is especially true when l'Hôpital's Rule is difficult or otherwise loops, especially with roots, e.g.  $\lim_{x \rightarrow \infty} \frac{x+1}{\sqrt{x^2+4}}$ .
- Know when l'Hôpital's Rule does not apply, e.g.  $\lim_{x \rightarrow \infty} \frac{x + \sin x}{x + \cos x}$ , and why l'Hôpital's Rule does not apply.

**Mean Value Theorem:**

- Know the statement of the Mean Value Theorem (MVT).
- Be able to check whether the MVT can be applied to a function on a given interval.
- Be able to apply the MVT to a function on a given interval.
- Know 'real life' applications and statements of the MVT.
- Be able to find values  $c$  that satisfy the statement of the MVT on a given interval.
- Be able to use the MVT to prove inequalities.
- Be able to use the MVT to bound function values on a given interval.

**Derivatives:**

- Know the statements of the First and Second Derivative Tests.
- Know how to apply the First and Second Derivative Tests.
- Be able to find critical values for functions and classify whether they are a maximum, minimum, or neither.

- Be able to use the derivative to find the intervals where a function is increasing and decreasing.
- Be able to use the second derivative to find the intervals where a function is concave and convex.
- Be able to find points of inflection.
- Be able to find the maximum/minimum values of a function on a closed interval, i.e. find max/min of  $f(x)$  on  $[a, b]$ . [The usual work along with comparing the endpoint values.]
- Given a graph of  $f(x)$ , be able to find where...
  - intervals where  $f(x)$  is increasing/decreasing
  - intervals where  $f(x)$  is concave/convex
  - points of maximum/minimum
  - points of inflection
  - Sketch a graph of  $f'(x)$
- Given a graph of  $f'(x)$ , be able to find where...
  - intervals where  $f(x)$  is increasing/decreasing
  - intervals where  $f(x)$  is concave/convex
  - points of maximum/minimum
  - points of inflection
  - Sketch a possible graph of  $f(x)$
  - Sketch a graph of  $f''(x)$
- Given a function  $f(x)$ , be able to provide a detailed sketch of  $f(x)$ , including finding: vertical/horizontal asymptotes, maximum/minimum values, points of inflection, intervals of increasing/decreasing, intervals where  $f(x)$  is concave/convex

#### Optimization:

- Be able to identify quantities to be optimized in a given problem.
- Be able to give rough sketches of various scenarios in an optimization problem.
- Know various area & volume formulas:
  - Volume Cube:  $V = s^3$
  - Surface Area Cube:  $SA = 6s^2$
  - Volume Rectangular Prism:  $V = lwh$
  - Surface Area Rectangular Prism:  $SA = 2(lw + lh + hw)$
  - Area Square:  $A = s^2$
  - Perimeter Square:  $L = 4s$
  - Area Rectangle:  $A = lw$
  - Perimeter Rectangle:  $L = 2l + 2w$
  - Area Circle:  $A = \pi r^2$
  - Circumference Circle:  $C = 2\pi r$

- Volume Right Circular Cone:  $V = \frac{\pi r^2 h}{3}$
- Surface Area Right Circular Cone:  $S = \pi r \sqrt{r^2 + h^2} + \pi r^2$
- Volume Cylinder:  $V = \pi r^2 h$
- Surface Area Cylinder:  $S = 2\pi r h + 2\pi r^2$
- Volume Sphere:  $V = \frac{4}{3}\pi r^3$
- Surface Area Sphere:  $S = 4\pi r^2$
- Area Triangle:  $A = \frac{bh}{2}$

- Be able to find maximum or minimum values of an optimized quantity in an applied problem.
- Be able to confirm a value is a maximum/minimum value in an applied problem.