l'Hôpital's Rule

Problem 1: Evaluate the following limits *without using l'Hôpital's Rule*:

(a) $\displaystyle\lim_{x\to\infty} \frac{2x^2+4x+5}{3x^2+x-1}$

(b) $\displaystyle\lim_{x\to-\infty} \frac{x^3-x+2}{6x^2+5x+7}$

(c) $\displaystyle\lim_{x\to\infty} \frac{4x^2+3x^2+1}{x^3+x+1}$

(d) $\displaystyle\lim_{x\to-\infty} \frac{5x+6}{\sqrt{3x^2+9}}$

(e) $\displaystyle\lim_{x\to\infty} \frac{2x^2+x+1}{\sqrt[3]{x^3+1}}$

(f) $\displaystyle\lim_{x\to\infty} \frac{1-x}{\sqrt[3]{x^4+x+1}}$

Problem 2: Find the following limits:

(a) $\displaystyle\lim_{x\to\infty} \frac{6x+3}{1-7x}$

(b) $\displaystyle\lim_{x\to-2^+} \frac{3x+6}{\ln(3x+7)}$

(c) $\displaystyle\lim_{x\to\infty} \frac{5^x}{x^2+x+1}$

(d) $\displaystyle\lim_{x\to 0} \frac{\sin(2x)}{\sin(3x)}$

(e) $\displaystyle\lim_{x\to\infty} \frac{\ln(5x)}{\sqrt{3x}}$

(f) $\displaystyle\lim_{x\to 0} \frac{1-\cos(5x)}{3x^2}$

Problem 3: Find the following limits:

(a) $\displaystyle\lim_{x\to 1} \left(\frac{1}{\ln x} - \frac{1}{x-1}\right)$

(b) $\displaystyle\lim_{x\to 1} \frac{x-1}{x-1+x\ln x}$

(c) $\displaystyle\lim_{x\to 0^+} x\ln x^3$

(d) $\displaystyle\lim_{x\to\infty} x\sin(1/x)$

(e) $\displaystyle\lim_{x\to\infty} x^2\,3^{-x}$

(f) $\displaystyle\lim_{x\to 1^+} \left(\frac{4}{x^2-1} - \frac{2}{x-1}\right)$

(g) $\displaystyle\lim_{x\to\infty} \big(\ln(6x+1)-\ln(2x+7)\big)$

(h) $\displaystyle\lim_{x\to 3^+} \left(\frac{1}{x^2-9} - \frac{\sqrt{x-2}}{x^2-9}\right)$

Problem 4: Find the following limits:

(a) $\displaystyle\lim_{x\to 0^+} (2x)^x$

(b) $\displaystyle\lim_{x\to 0^+} x^{2x}$

(c) $\displaystyle\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x$

(d) $\displaystyle\lim_{x\to\infty} x^{1/x}$

(e) $\displaystyle\lim_{x\to\infty} (e^x+x)^{2/x}$

(f) $\displaystyle\lim_{x\to 1^+} x^{1/(x-1)}$

Problem 5: Explain why l'Hôpital's Rule does not apply to the limit $\displaystyle\lim_{x\to\infty} \frac{x^2+\sin(2x)}{2x^2-\cos(3x)}$. Find the limit.

Problem 6: Explain why l'Höpital's Rule does not apply to the limit $\displaystyle\lim_{x\to 0} \frac{1/x+\sin(1/x)}{1/x}$. Find the limit.

Problem 7: Explain why l'Höpital's Rule does not apply to the limit $\lim_{x\to\infty} \dfrac{1+\sin(1/x)}{x}$. Find the limit.

Problem 8: Use l'Höpital's Rule to compute $\lim_{x\to\infty}\left(\sqrt{x^2+2x+1}-x\right)$. [HINT: Factor out an x first.]

Problem 9: Find the following limits:

(a) $\lim_{x\to\infty} x^{\sin(1/x)}$

(b) $\lim_{x\to\infty} \dfrac{\sqrt[3]{x}}{\ln x}$

(c) $\lim_{x\to\pi/2} \cos(x)\ln(x-\pi/2)$

(d) $\lim_{x\to 0}\left(\csc x - \dfrac{1}{x}\right)$

(e) $\lim_{x\to 0} \dfrac{e^x - x - 1}{\cos x - 1}$

(f) $\lim_{x\to 0} (\csc x)^{\cos x}$

Problem 10: Find the following limits:

(a) $\lim_{x\to 0^+} \dfrac{x\cot x}{e^x - 1}$

(b) $\lim_{x\to 4} \dfrac{\sqrt{x+5}-3}{x-4}$

(c) $\lim_{x\to 0^+} \dfrac{\sin(5x)\cot(3x)}{\ln(\cos x)}$

(d) $\lim_{x\to 0} x^2 \ln x$

(e) $\lim_{x\to 0} \dfrac{\arcsin x}{x^2 \csc x}$

(f) $\lim_{x\to 0^+} \left(\dfrac{\sin x}{x}\right)^{1/x^2}$

Problem 11: Find the following limits:

(a) $\lim_{x\to 0} \dfrac{2\cos^{-1} x - \pi}{2x}$

(b) $\lim_{x\to 1} \dfrac{\sqrt{8+x}-3x^{1/3}}{x^2-3x+2}$

(c) $\lim_{x\to\infty} (x - \ln x)$

(d) $\lim_{x\to\infty} \dfrac{x^2+e^x}{x^2+e^{3x}}$

(e) $\lim_{x\to 6} \dfrac{x-6}{\ln x - \ln 6}$

(f) $\lim_{x\to 2} \dfrac{2x^2-x-6}{x^2-4}$

Problem 12: How should one compute the following limit:

$$\lim_{n\to-\infty} \frac{n^{11}+3n^9-n^8+5n^6-4n^3+2n-1}{16n^{10}+n^7-143n^6+67n^5-4n^3+16n-12}$$

Use your method to compute the limit.

Problem 13: Is l'Hôpital's Rule the 'easiest' way of computing the following limit:

$$\lim_{x\to 0}\left(\frac{1}{2x}-\frac{1}{x}\right)$$

Compute the limit.

Problem 14: Compute the limit

$$\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}$$

What is wrong with using l'Hôpital's Rule for this limit?

Mean Value Theorem

Problem 15: A highway camera clocks you going through the a toll booth at 2:00 PM. Another camera captures you going through another toll booth 70 mi away at 3:00 PM. If the speed limit is never over 65 mph, should you be fined for speeding? What mathematics could the state use to justify any possible fine?

Problem 16: If a car is driving down a highway towards a destination 400 mi away, always traveling between 55 mph and 70 mph, at least how long does the trip have to last? What about at most how long? Justify your response.

Problem 17: Can the Mean Value Theorem be applied to $f(x) = \dfrac{x+2}{x-1}$ on the interval $[0,1]$? Why or why not. Can it be applied to $f(x)$ on the interval $[5,10]$? Why or why not.

Problem 18: Show that $g(x) = x^3 + 2x^2 - x - 1$ satisfies the hypotheses of the Mean Value Theorem on the interval $[-2,1]$. Find any $c \in (-2,1)$ which satisfy the theorem.

Problem 19: If the Mean Value Theorem applies to $f(x)$ on $[a,b]$, show that if $f'(x) > 0$ on $[a,b]$, then $f(x)$ is increasing on $[a,b]$. Show that if $f'(x) < 0$ on $[a,b]$, then $f(x)$ is decreasing on $[a,b]$.

Problem 20: Show that if $f'(x) = 0$ on an interval $[a,b]$, then $f(x)$ is constant on $[a,b]$. Use this to show that if there are functions $f(x), g(x)$ with $f'(x) = g'(x)$ on an interval $[a,b]$, then $f(x) = g(x) + C$ for some constant C.

Problem 21: If $f(x)$ is continuously differentiable on $[1,3]$, $f(3) = 5$, and $1 \le f'(x) \le 2$ for all $x \in [1,3]$, find the smallest possible interval I for which you can guarantee $f(1) \in I$.

Problem 22: Show that $g(x) = (x+1)^3$ satisfies the hypothesis of the Mean Value Theorem on the interval $[-1,1]$. Find all possible $c \in (-1,1)$ satisfying the statement of the theorem.

Problem 23: Show that $y = x \ln x$ satisfies the hypotheses of the Mean Value Theorem on the interval $[1,3]$. Find any $c \in (1,3)$ which satisfy the conditions of the theorem.

Problem 24: Show that $|\sin a - \sin b| \le |a - b|$ for all $a \ne b$. [HINT: Apply the MVT to $f(x) = \sin x$ on $[a,b]$.]

Problem 25: Show that if $0 < x < y$, then $\sqrt{y} - \sqrt{x} < \dfrac{y-x}{2\sqrt{x}}$. [HINT: Apply the MVT to $f(x) = \dfrac{y-x}{2\sqrt{x}} - (\sqrt{y} - \sqrt{x})$ on $[x,y]$. However, there is a method without the MVT. Can you find it?]

Problem 26: Show that $x^5 + 4x = 1$ has exactly one solution. [HINT: Show that it has at least one using IVT, then show this is the only one using MVT.]

Problem 27: Suppose f has two continuous derivatives on $[0,6]$. If $f(0) = 1$, $f(3) = 4$, $f(6) = 7$, show there is a point $a \in (0,6)$ so that $f''(a) = 0$.

Problem 28: For $a > 0$, show that $\dfrac{a}{a^2+1} < \arctan a < a$. [HINT: Apply the MVT to $f(x) = \arctan x$ on $[0,N]$ for any $N > 0$ and bound the derivative.]

Problem 29: Let $0 < a < b$ and $n \geq 2$ be an integer. Show that

$$na^{n-1}(b-a) \leq b^n - a^n \leq nb^{n-1}(b-a).$$

[HINT: Apply the MVT to $f(x) = x^n$ on $[a, b]$. What do you know about $f(x)$? What does this mean about $f'(x)$?]

Increasing/Decreasing & Concavity

Problem 30: Find the intervals where the function $f(x) = 4x^3 + 8x^2 - 3x + 1$ is increasing or decreasing. Identify the x-value for any max/mins. Also, find the intervals where the function is concave and convex. Identity the x-value of any points of inflection.

Problem 31: Find the intervals where the function $f(x) = 1 + x - 2x^2$ is increasing or decreasing. Identify the x-value for any max/mins. Also, find the intervals where the function is concave and convex. Identity the x-value of any points of inflection.

Problem 32: Find the intervals where the function $f(x) = x^4 - 18x^2 + 5$ is increasing or decreasing. Identify the x-value for any max/mins. Also, find the intervals where the function is concave and convex. Identity the x-value of any points of inflection.

Problem 33: Find the intervals where the function $f(x) = 25x^2 - 4x^{5/2}$ is increasing or decreasing. Identify the x-value for any max/mins. Also, find the intervals where the function is concave and convex. Identity the x-value of any points of inflection.

Problem 34: Find the intervals where the function $f(x) = x^2 - 3x^{4/3}$ is increasing or decreasing. Identify the x-value for any max/mins. Also, find the intervals where the function is concave and convex. Identity the x-value of any points of inflection.

Problem 35: Find the intervals where the function $f(x) = \dfrac{1}{x^2 + 4}$ is increasing or decreasing. Identify the x-value for any max/mins. Also, find the intervals where the function is concave and convex. Identity the x-value of any points of inflection.

Problem 36: Find the intervals where the function $f(x) = \dfrac{x}{x^2 + 1}$ is increasing or decreasing. Identify the x-value for any max/mins. Also, find the intervals where the function is concave and convex. Identity the x-value of any points of inflection.

Problem 37: Find the intervals where the function $f(x) = \dfrac{1 - 2x}{x + 3}$ is increasing or decreasing. Identify the x-value for any max/mins. Also, find the intervals where the function is concave and convex. Identity the x-value of any points of inflection.

Problem 38: Find the intervals where the function $f(x) = \cos^2 x$ is increasing or decreasing on the interval $[0, 2\pi]$. Identify the x-value for any max/mins on this interval. Also, find the where the function is concave and convex on this interval. Identity the x-value of any points of inflection in this interval.

Problem 39: Find the intervals where the function $f(x) = \log(x^2 + 5x + 6)$ is increasing or decreasing. Identify the x-value for any max/mins. Also, find the intervals where the function is concave and convex. Identity the x-value of any points of inflection.

Problem 40: Suppose that $C(x)$ is the cost of producing x items of a certain good. The average cost of producing each item is then $A(x) := \dfrac{C(x)}{x}$. If $C(x)$ is concave up, show that the average cost is minimized at a production level x_0 where $A(x_0) = C'(x_0)$, i.e. the average cost is minimized at a production level where the average cost equals the minimal cost. Show also that the tangent lint through the origin and this point on the cost curve is tangent to the curve $C(x)$.

Max/Mins & Optimization on Intervals

Problem 41: Find the maximum and minimum values of $f(x) = \sin x + \cos x$ on the interval $[-2, 3]$. What are the absolute max and min on this interval?

Problem 42: Find the maximum and minimum values of $f(x) = 8x - x^2$ on the interval $[0, 5]$. What are the absolute max and min on this interval?

Problem 43: Find the maximum and minimum values of $f(x) = \dfrac{x^2 + 5}{x - 2}$ on the interval $[-2, 6]$. What are the absolute max and min on this interval?

Problem 44: Find the maximum and minimum values of $f(x) = x - \dfrac{2x}{x + 1}$ on the interval $[0, 2]$. What are the absolute max and min on this interval?

Problem 45: Find the maximum and minimum values of $f(x) = \sin x + \cos x$ on the interval $[0, \frac{\pi}{2}]$.

Problem 46: Find the maximum and minimum values of $f(x) = xe^{-8x^2}$ on the interval $[-2, -1]$. What are the absolute max and min on this interval?

Problem 47: Find the maximum and minimum values of $f(x) = x^7 - \frac{5}{7}x^5$ on the interval $[-1, 1]$. What are the absolute max and min on this interval?

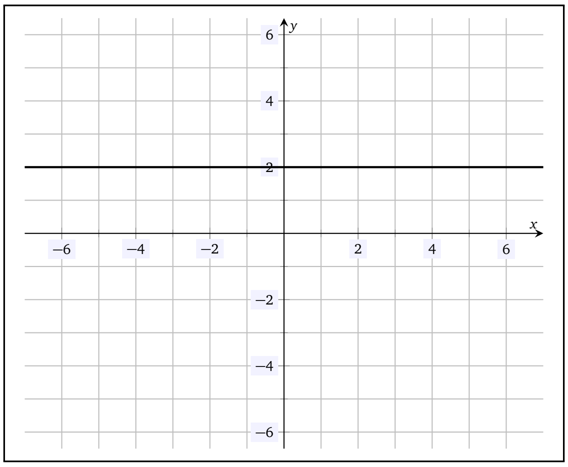
Problem 48: Find the maximum and minimum values of $f(x) = \log(x + 1) + \log(x + 4)$ on the interval $[0, 1]$. What are the absolute max and min on this interval?

Problem 49: Find the maximum and minimum values of $f(x) = x^2 e^{x/2} - 2$ on the interval $[-2, 1]$. What are the absolute max and min on this interval?

Problem 50: Find the maximum and minimum values of $f(x) = \sin^2 x$ on the interval $[0, 2\pi]$. What are the absolute max and min on this interval?

Derivatives & Graphs

Problem 51: Let $f(x)$ be a twice differentiable function. The function $f'(x)$ is plotted below.



(a) Where is $f(x)$ increasing? Where is $f(x)$ decreasing?

(b) Does $f(x)$ have any max/mins? If so, where.

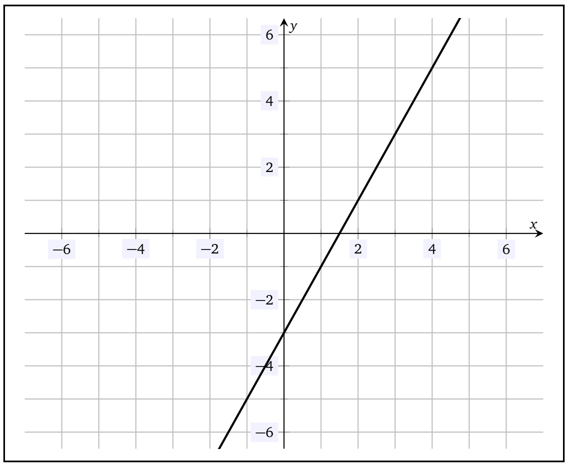
(c) Where is $f(x)$ concave? Where is $f(x)$ convex?

(d) Does $f(x)$ have any points of inflection?

(e) Assuming $f''(x)$ exists, where does it have max/mins?

(f) Try to sketch a possible $f(x)$ on the plot.

Problem 52: Let $f(x)$ be a twice differentiable function. The function $f'(x)$ is plotted below.



(a) Where is $f(x)$ increasing? Where is $f(x)$ decreasing?

(b) Does $f(x)$ have any max/mins? If so, where.

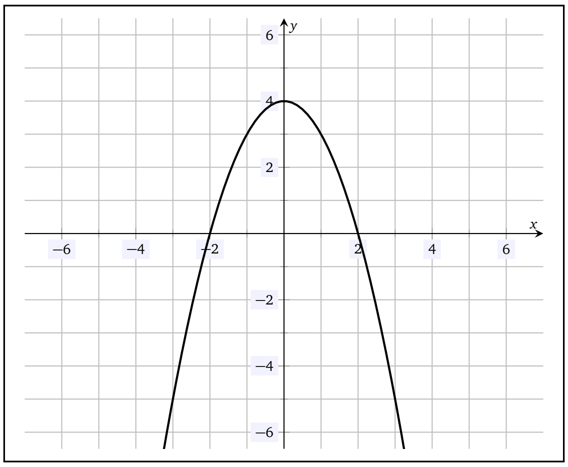
(c) Where is $f(x)$ concave? Where is $f(x)$ convex?

(d) Does $f(x)$ have any points of inflection?

(e) Assuming $f''(x)$ exists, where does it have max/mins?

(f) Try to sketch a possible $f(x)$ on the plot.

Problem 53: Let $f(x)$ be a twice differentiable function. The function $f'(x)$ is plotted below.



(a) Where is $f(x)$ increasing? Where is $f(x)$ decreasing?

(b) Does $f(x)$ have any max/mins? If so, where.

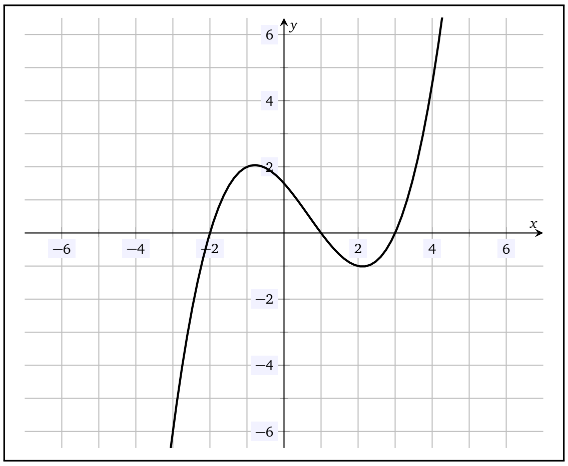
(c) Where is $f(x)$ concave? Where is $f(x)$ convex?

(d) Does $f(x)$ have any points of inflection?

(e) Assuming $f''(x)$ exists, where does it have max/mins?

(f) Try to sketch a possible $f(x)$ on the plot.

Problem 54: Let $f(x)$ be a twice differentiable function. The function $f'(x)$ is plotted below.



(a) Where is $f(x)$ increasing? Where is $f(x)$ decreasing?

(b) Does $f(x)$ have any max/mins? If so, where.

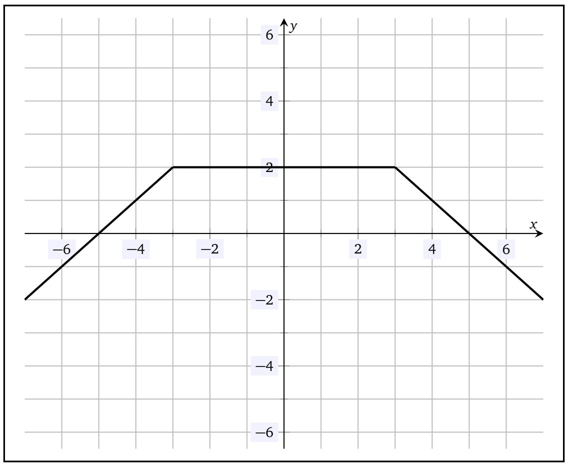
(c) Where is $f(x)$ concave? Where is $f(x)$ convex?

(d) Does $f(x)$ have any points of inflection?

(e) Assuming $f''(x)$ exists, where does it have max/mins?

(f) Try to sketch a possible $f(x)$ on the plot.

Problem 55: Let $f(x)$ be a twice differentiable function. The function $f'(x)$ is plotted below.



(a) Where is $f(x)$ increasing? Where is $f(x)$ decreasing?

(b) Does $f(x)$ have any max/mins? If so, where.

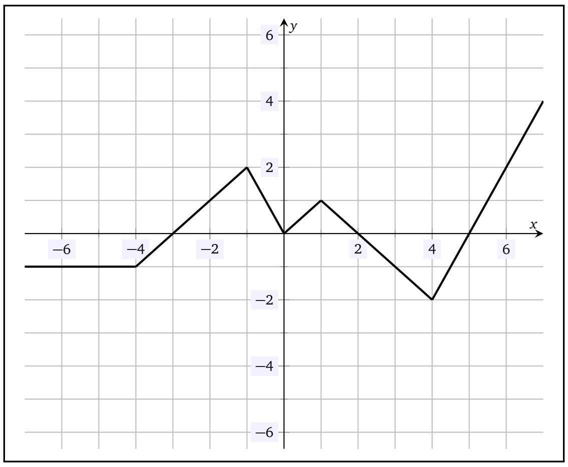
(c) Where is $f(x)$ concave? Where is $f(x)$ convex?

(d) Does $f(x)$ have any points of inflection?

(e) Assuming $f''(x)$ exists, where does it have max/mins?

(f) Try to sketch a possible $f(x)$ on the plot.

Problem 56: Let $f(x)$ be a twice differentiable function. The function $f'(x)$ is plotted below.



(a) Where is $f(x)$ increasing? Where is $f(x)$ decreasing?

(b) Does $f(x)$ have any max/mins? If so, where.

(c) Where is $f(x)$ concave? Where is $f(x)$ convex?

(d) Does $f(x)$ have any points of inflection?

(e) Assuming $f''(x)$ exists, where does it have max/mins?

(f) Try to sketch a possible $f(x)$ on the plot.

Curve Sketching

Problem 57: Sketch the curve $f(x) = \dfrac{2(x^2-9)}{x^2-4}$.

Problem 58: Sketch the curve $f(x) = \dfrac{x^2-2x+4}{x-2}$.

Problem 59: Sketch the curve $f(x) = \dfrac{1}{x(x-1)}$.

Problem 60: Sketch the curve $f(x) = x^2e^{-2x}$.

Problem 61: Sketch the curve $f(x) = \dfrac{x^3+8}{x}$.

Problem 62: Sketch the curve $f(x) = x(x-4)^3$.

Problem 63: Sketch the curve $f(x) = \dfrac{\cos x}{1+\sin x}$.

Problem 64: Sketch the curve $f(x) = \dfrac{x}{x^2+16}$.

Problem 65: Sketch the curve $f(x) = x(12-x)^{1/3}$.

Problem 66: Sketch the curve $f(x) = x^4-4x^3$.

Problem 67: Sketch the curve $f(x) = x^3-\frac{3}{2}x^2$.

Problem 68: Sketch the curve $f(x) = x-2\ln(x^2+1)$.

Problem 69: Sketch the curve $f(x) = 2x-3x^{2/3}$.

Problem 70: Sketch the curve $f(x) = \dfrac{9(x^2-3)}{x^3}$.

Problem 71: Sketch the curve $f(x) = x^3-6x^2+5$.

Problem 72: Sketch the curve $f(x) = \dfrac{1-x^2}{2-x}$.

Problem 73: Sketch the curve $f(x) = x^4-3x^3+4x$.

Problem 74: Sketch the curve $f(x) = (x^2-9x)^{1/3}$.

Problem 75: Sketch the curve $f(x) = x-\log(x^2+1)$.

Problem 76: Sketch the curve $f(x) = \sin x+\cos x$ on the interval $[0, 2\pi]$.

Optimization

Problem 77: Find a positive number such that the sum of the number and its reciprocal is as small as possible. Is there a largest sum? Explain.

Problem 78: Find two numbers whose sum is 100 and product is maximal. Are there two such numbers whose product is minimal? Explain.

Problem 79: A 10 m tube is to be bent into an 'L' to connect two pipes. Due to pressure concerns, the ends need to be kept as close as possible for observation. Where should the pipe be bent?

Problem 80: Assume you are constructing a box with square base. If the volume must be 10 m^3, what should the dimensions be to minimize the surface area? What is the surface area must be 10 m^2. What are the dimension of the box maximizing the volume?

Problem 81: What is the maximum area of a rectangle which can be inscribed in a right triangle with legs 5 and 12?

Problem 82: A holiday gift wrapper is making open boxes from flat pieces of cardboard by cutting out identical squares in each corner and folding the flaps upwards. If the piece of cardboard is 24 in by 12 in, what is the size of the cut squares that maximizes the volume of the folded box?

Problem 83: What are the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius $r = 9$?

Problem 84: Suppose someone wants to construct an open box with base length three times its width. The material used to build the material used to make the bottom costs \$10/ft^2 and the sides cost \$6/ft^2. If the box is to have a volume 50 ft^3, find the dimensions of the box that will minimize the cost.

Problem 85: A fence to hold cattle is to be constructed against a large cliff, so that one side of the fence will not need to be constructed. To either side of the fence is a highway, so that the fence there will be more secure. If the secure sides fencing costs \$6/ft and any other fencing cost \$4/ft, what is the area of the largest holding area that can be constructed?

Problem 86: If you are going to construct an isosceles triangle with leg lengths both 5 ft, what should the angle between these two legs be to maximize the area of the triangle?

Problem 87: Find the point on the curve $y = x^2 + x + 1$ closest to the point $(4, 0)$.

Problem 88: A manufacturer is going to make a cylindrical can with an open top which will hold 1 L of liquid. Determine the dimensions of the can which will maximize the volume of the can.

Problem 89: A manufacturer wants to design an open box with a square base and total surface area 108 ft^2. What are the dimensions of the box with largest possible volume?

Problem 90: Find the point on the curve $y = -x^2 - 2x - 1$ closest to the point $(-1, 2)$.

Problem 91: A printer is going to make a poster having a total area of 200 in^3 with 1 in margins, a 2 in margin on the top, and 1.5 in margins on the bottom. What dimensions of the poster will give the largest area upon which to print the poster?

Problem 92: Assuming a classical model of the atom, Niels Bohr was able to show that the energy of a hydrogen atom with separation r between the proton and the election is given by

$$E(r) = \frac{\hbar^2}{2m_e r^2} - \frac{e^2}{4\pi\epsilon_0 r}$$

where $\hbar$ is the reduced Planck's constant (Dirac constant), m_e is the mass of the election, e is the charge of an electron, and ϵ_0 is permittivity of free space. The Bohr radius for the hydrogen atom, denoted r_{Bohr}, is the radius at which $E(r)$ is minimal. Furthermore, r_{Bohr} is approximately the expected distance between the proton and the electron in the ground state. Show that

$$r_{\text{Bohr}} = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2}$$

Problem 93: The Morse potential - which measures the potential energy of diatomic molecules. This model is often better (certainly easier to use) than the quantum harmonic oscillator in approximating vibrational structures for diatomic molecules. The Morse potential is given by

$$V(r) = D_e\left(1 - e^{-a(r-r_e)}\right)^2$$

where r is the separation of the atoms, r_e is the equilibrium separation, and D_e is the potential well depth, and $a = \sqrt{\frac{k_e}{2D_e}}$ where k_e is force at the bottom of the well. Using this model, what is the separation giving the minimal potential energy? Could you get it by inspection alone?

Problem 94: Find the largest isosceles triangle which can be inscribed in a circle of radius r.

Problem 95: Find the rectangle with largest area whose vertices lie on the positive x and y-axes and the curve $y = \dfrac{12-x}{x+4}$.

Problem 96: A box of fixed volume 64 m^3 has a square bottom. The box is made of two different materials. The bottom material costs \$30/m^2 while the sides cost \$40/m^2. What are the dimensions of the box which minimize the total cost?

Problem 97: A window is being framed. If the window has a rectangular base and a semicircular top. If there is 12 m of framing materials available, what dimensions the window will allow the most light to pass through?

Problem 98: You want to cross a river. You are going to launch a boat from a point along the river, which is 5 km directly across from the opposite bank. The endpoint of the journey is at a point 3 km downstream. If you can row 4 km/hr and run 5 km/hr, how far down the river should you land the boat to minimize the time it takes to reach your destination, assuming you can only row in straight lines.

Problem 99: Determine the cylinder with largest volume which can be inscribed in a right circular cone of height 8 cm and base 4 cm.

Problem 100: A livestock researcher is conducting a breeding project. Accordingly, the most separate out the livestock. If they are to build a large rectangular pen with two straight fences dividing the pen into three enclosures and only have 3,000 m of fencing, what is the maximum total area for the pen?

Problem 101: An 8 ft high fence is parallel to the wall of a building with the bottom of the wall 1 ft from the building. What is the shortest plank which can be leaned against the fence and reach over the fence and touch the building?

Problem 102: What points on the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ are furthest from the point $(2,0)$?

Problem 103: If two hallways of widths 4 ft and 32 ft meet at a corner, what is the longest couch, pole, or other large object that can round this corner?