Quiz 1: Complete the following chart:

Solution:

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undef.

Quiz 2: Circle whether the following are True or False:

Solution:

(i) If $\lim_{x\to 3} f(x)$ exists, then $\lim_{x\to 3^-} f(x)$ and $\lim_{x\to 3^+} f(x)$ *both* exist. **[True]** False

(ii) If $\lim_{x\to -1^+} g(x)$ and $\lim_{x\to -1^-} g(x)$ *both* exist, then $\lim_{x\to -1} g(x)$ exists. True **[False]**

x

Quiz 3: Compute the following limit: $\lim_{h\to 0} \frac{3(h+1)^2-3}{h}$

Solution:

$$\lim_{h\to 0} \frac{3(h+1)^2-3}{h} = \lim_{h\to 0} \frac{3(h^2+2h+1)-3}{h} = \lim_{h\to 0} \frac{3h^2+6h+3-3}{h} = \lim_{h\to 0} \frac{3h^2+6h}{h} = \lim_{h\to 0} \frac{\cancel{h}(3h+6)}{\cancel{h}}$$
$$= \lim_{h\to 0}(3h+6) = 6$$

Quiz 4: Compute the following limit: [You do not need to show your work.]

$$\lim_{x\to\infty} \frac{2x^3 - 3x^2 + x - 7}{3x^3 + x^2 - 1} =$$

Solution:

$$\lim_{x\to\infty} \frac{2x^3 - 3x^2 + x - 7}{3x^3 + x^2 - 1} = \frac{2}{3}$$

———————————— x ————————————

Quiz 5: Let $f(x) := \dfrac{2x^2 - \sin x}{x - 3}$. Mark whether $f(x)$ is continuous on the following intervals; use '✓' to indicate that it is continuous and '✗' to indicate that it is not continuous.

(i) ______: $\mathbb{R}$

(ii) ______: $[-1, 1]$

(iii) ______: $[-5, 5]$

(iv) ______: $[0, 1]$

Solution:

(i) __✗__: $\mathbb{R}$

(ii) __✓__: $[-1, 1]$

(iii) __✗__: $[-5, 5]$

(iv) __✓__: $[0, 1]$

———————————— x ————————————

Quiz 6: A student is attempting to prove that there is a solution to $x^3 + 2x - 1 = x$. The student completes the problem in the steps shown below. Place a '✓' in the step where there is a mistake, or place a '✓' in the blank space in (v) to indicate steps (i)–(iv) are correct.

(i) _____: The equation has a solution if and only if $x^3 + x - 1 = 0$.

(ii) _____: The function $f(x) := x^3 + x - 1$ is continuous.

(iii) _____: $f(0) < 0 < f(1)$

(iv) _____: By the Intermediate Value Theorem, there is a $c \in (0, 1)$ so that $c^3 + c - 1 = 0$, i.e. $c^3 + 2c - 1 = c$.

(v) __✓__: The solution is correct.

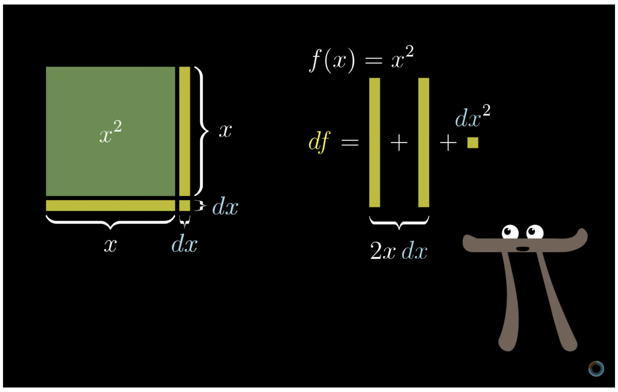
Quiz 7: Use the Squeeze Theorem to prove $\lim_{x\to 0} x^2 \sin\left(\frac{1}{|x|}\right) = 0$.

Solution: We know $-1 \le \sin x \le 1$. Therefore, $-x^2 \le x^2 \sin\left(\frac{1}{|x|}\right) \le x^2$. But we know that $\lim_{x\to 0}(-x^2) = \lim_{x\to 0} x^2 = 0$. Therefore by Squeeze Theorem,

$$\lim_{x\to 0} x^2 \sin\left(\frac{1}{|x|}\right) = 0.$$

□

x



Quiz 8: The figure above is a graphical reasoning for the derivative of what function?

Solution: $f(x) = x^2$ [To rewatch the video, visit https://www.youtube.com/watch?v=S0_qX4VJhMQ.]

x

Quiz 9: Let $f(x) = 2x^2 + x + 1$. Use the definition of the derivative to find $f'(1)$.

Solution:

$$\begin{aligned} f'(1) :&= \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} \\ &= \lim_{h\to 0} \frac{[2(1+h)^2 + (1+h) + 1] - 4}{h} \\ &= \lim_{h\to 0} \frac{2(h^2 + 2h + 1) + h - 2}{h} \\ &= \lim_{h\to 0} \frac{2h^2 + 5h + 2 - 2}{h} \\ &= \lim_{h\to 0}(2h + 5) = 5 \end{aligned}$$

Quiz 10: Find the following derivatives:

(a) $\dfrac{d}{dx}\ \sin x = \cos x$

(b) $\dfrac{d}{dx}\ x^3 = 3x^2$

(c) $\dfrac{d}{dx}\ \cos x = -\sin x$

(d) $\dfrac{d}{dx}\ \pi^2 = 0$

(e) $\dfrac{d}{dx}\ e^x = e^x$

(f) $\dfrac{d}{dx}\ \tan^{-1} x = \dfrac{1}{1+x^2}$

x

Quiz 11: Find the tangent line to the function $f(x) = \dfrac{x^2 \sin(x^2-1)+6}{x^2+1}$ at $x = 1$.

Solution:

$$\frac{d}{dx}\left[\frac{x^2 \sin(x^2-1)+6}{x^2+1}\right] = \frac{(x^2+1)(2x\sin(x^2-1) + x^2 \cdot \cos(x^2-1)\cdot 2x) - (2x)(x^2\sin(x^2-1)+6)}{(x^2+1)^2}$$

$$\frac{d}{dx}\left[\frac{x^2 \sin(x^2-1)+6}{x^2+1}\right]_{x=1} = \frac{(1+1)(2\sin(0)+\cos(0)\cdot 2) - 2(\sin(0)+6)}{2^2}$$

$$= \frac{2\cdot(0+2)-2(0+6)}{4} = \frac{4-12}{4} = \frac{-8}{4} = -2$$

We also have $f(1) = \dfrac{1\cdot\sin(0)+6}{2} = \dfrac{6}{2} = 3$. Then

$$y = y_0 + m(x-x_0)$$
$$y = 3 - 2(x-1)$$
$$y = 3 - 2x + 2$$
$$y = 5 - 2x$$

Quiz 12: Find $\frac{dy}{dx}$ for the curve $x^2 + xy + y^2 = 8$.

Solution:

$$\begin{aligned} x^2 + xy + y^2 &= 8 \\ \frac{d}{dx}[x^2 + xy + y^2] &= \frac{d}{dx}\, 8 \\ 2x + (y + xy') + 2yy' &= 0 \\ xy' + 2yy' &= -2x - y \\ y'(x + 2y) &= -(2x + y) \\ \frac{dy}{dx} &= -\frac{y + 2x}{x + 2y} \end{aligned}$$

———————— x ————————

Quiz 13: Compute the following limit: $\lim_{x \to 0} \frac{3 - \sqrt{x+9}}{x}$.

Solution:

$$\begin{aligned} \lim_{x \to 0} \frac{3 - \sqrt{x+9}}{x} &= \lim_{x \to 0} \frac{3 - \sqrt{x+9}}{x} \cdot \frac{3 + \sqrt{x+9}}{3 + \sqrt{x+9}} \\ &= \lim_{x \to 0} \frac{9 - (x+9)}{x(3 + \sqrt{x+9})} \\ &= \lim_{x \to 0} \frac{-x}{x(3 + \sqrt{x+9})} \\ &= \lim_{x \to 0} \frac{-1}{3 + \sqrt{x+9}} \\ &= -\frac{1}{3 + \sqrt{0+9}} = -\frac{1}{6} \end{aligned}$$

Quiz 14: Find $\frac{d^2y}{dx^2}$, where $y^2 - x^2y = 5$.

Solution: We have

$$\begin{aligned} y^2 - x^2y &= 5 \\ \frac{d}{dx}\left[y^2 - x^2y\right] &= \frac{d}{dx}\,5 \\ 2yy' - (2xy + x^2y') &= 0 \\ 2yy' - 2xy - x^2y' &= 0 \\ y'(2y - x^2) &= 2xy \\ \frac{dy}{dx} &= \frac{2xy}{2y - x^2} \end{aligned}$$

Then we must have

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{d}{dx}\,\frac{dy}{dx} \\ &= \frac{d}{dx}\left[\frac{2xy}{2y - x^2}\right] \\ &= \frac{(2y - x^2)(2y + 2xy') - (2y' - 2x)(2xy)}{(2y - x^2)^2} \\ &= \frac{(2y - x^2)\left(2y + 2x \cdot \frac{2xy}{2y - x^2}\right) - \left(2 \cdot \frac{2xy}{2y - x^2} - 2x\right)(2xy)}{(2y - x^2)^2} \end{aligned}$$

x

Quiz 15: Calculate the following limit: $\lim_{x \to 5} \frac{\frac{1}{5} - \frac{1}{x}}{5 - x}$.

Solution:

$$\lim_{x \to 5} \frac{\frac{1}{5} - \frac{1}{x}}{5 - x} = \lim_{x \to 5} \frac{\frac{x-5}{5x}}{5 - x} = \lim_{x \to 5} \frac{x - 5}{(5 - x)5x} = \lim_{x \to 5} \frac{-1}{5x} = -\frac{1}{25}$$

Quiz 16: Compute the following limit: $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right)$.

Solution: Since $-1 \le \sin\theta \le 1$ for all θ, we know that $-1 \le \sin\left(\frac{1}{x}\right) \le 1$ for all x. But multiplying by x^2, we then have

$$-x^2 \le x^2 \sin\left(\frac{1}{x}\right) \le x^2$$

But as we have $\lim_{x\to 0}(-x^2) = \lim_{x\to 0} x^2 = 0$. Therefore by Squeeze Theorem, we must have

$$\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0.$$

x

Quiz 17: Compute the following derivatives:

(i) $\frac{d}{dx} \sin^2(3x)\sec(2x) = 2\sin(3x)\cos(3x)3 \cdot \sec x + \sec^2(3x) \cdot \sec(2x)\tan(2x)\,2$

(ii) $\frac{d}{dx} \tan^{-1}(5x) = \frac{1}{1+(5x)^2} \cdot 5$

(iii) $\frac{d}{dx} \frac{(1-4x)^3}{1+e^x} = \frac{(1+e^x)\cdot 3(1-4x)^2\,4 - e^x \cdot (1-4x)^3}{(1+e^x)^2}$

(iv) $\frac{d}{dx} 5^x \log_4(1-x) = 5^x \ln 5 \cdot \log_4(1-x) + 5^x \cdot \frac{1}{(1-x)\ln 4} \cdot (-1)$

Quiz 18: Find $\frac{dy}{dx}$ for the curve $x + x^2y^3 = \sin(xy)$.

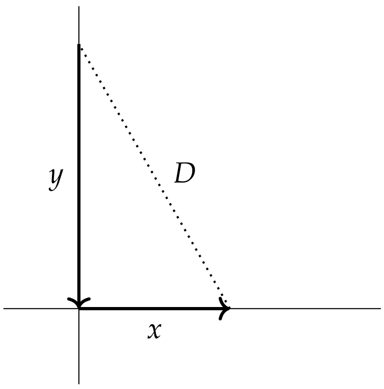
Solution:

$$\begin{aligned}
x + x^2y^3 &= \sin(xy) \\
\frac{d}{dx}\left[x + x^2y^3\right] &= \frac{d}{dx}\ \sin(xy) \\
1 + (2x \cdot y^3 + x^2 \cdot 3y^2y') &= \cos(xy) \cdot (1 \cdot y + x \cdot y') \\
1 + 2xy^3 + 3x^2y^2\,y' &= y\cos(xy) + x\cos(xy)\,y' \\
3x^2y^2\,y' - x\cos(xy)\,y' &= y\cos(xy) - 2xy^3 - 1 \\
\left(3x^2y^2 - x\cos(xy)\right)y' &= y\cos(xy) - 2xy^3 - 1 \\
\frac{dy}{dx} &= \frac{y\cos(xy) - 2xy^3 - 1}{3x^2y^2 - x\cos(xy)}
\end{aligned}$$

x

Quiz 19: An spider moves along the x-axis from left to right at 5 inches per second. A fly moves along the y-axis from up to down at 3 inches per second. At a certain instant, the spider is 4 inches to the right of the origin and the fly is 8 inches above the origin. At this instant, what is the rate of change of the distance between the spider and the fly? [Oh my!]

Solution:



Want $\frac{dD}{dt} = D'$. We know if $x = 4$, $y = 8$, that $D^2 = 4^2 + 8^2$ so that $D^2 = 80$, which means $D = 4\sqrt{5}$.

$$\begin{aligned}
D^2 &= x^2 + y^2 \\
\frac{d}{dt}\,D^2 &= \frac{d}{dt}\ \left[x^2 + y^2\right] \\
2DD' &= 2x \cdot x' + 2y \cdot y' \\
DD' &= x \cdot x' + y \cdot y' \\
\frac{dD}{dt} &= \frac{x \cdot x' + y \cdot y'}{D} \\
\frac{dD}{dt} &= \frac{4 \cdot 5 + 8 \cdot (-3)}{4\sqrt{5}} = -\frac{1}{\sqrt{5}} \text{ in/s}
\end{aligned}$$