Problem 1: Evaluate the following:

(a) $\displaystyle\int x^2\left(x+1-\frac{1}{x^2}+\frac{2}{x^3}\right)dx$

(b) $\displaystyle\int \frac{dx}{4+x^2}$

(c) $\displaystyle\int \sqrt[3]{(x+1)^5}\,dx$

(d) $\displaystyle\int \frac{x+5}{x-6}\,dx$

(e) $\displaystyle\int \sin^5\theta\,\cos^2\theta\,d\theta$

(f) $\displaystyle\int x^5\ln x\,dx$

(g) $\displaystyle\int (x^3-x+1)e^{2x}\,dx$

(h) $\displaystyle\int e^x\sin(3x)\,dx$

(i) $\displaystyle\int \frac{x-1-\sqrt{x}+\sqrt[3]{x}}{\sqrt{x}}\,dx$

(j) $\displaystyle\int \frac{dx}{9x^2+4}$

(k) $\displaystyle\int \sin\theta\,\sec^2\theta\,d\theta$

(l) $\displaystyle\int \sec^6\theta\,\tan^4\theta\,d\theta$

(m) $\displaystyle\int \arctan(1/x)\,dx$

(n) $\displaystyle\int 2x^2\sin(3x)\,dx$

(o) $\displaystyle\int \sin(2x)\cos(3x)\,dx$

(p) $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

(q) $\displaystyle\int x\cos(1-x^2)\,dx$

(r) $\displaystyle\int \sin^4\theta\,d\theta$

(s) $\displaystyle\int \frac{\ln x}{x^2}\,dx$

(t) $\displaystyle\int \frac{\sin x}{1+\sin x}\,dx$

Problem 2: Evaluate the following:

(a) $\displaystyle\int \frac{x-\sqrt{x}}{\sqrt{x}}\,dx$

(b) $\displaystyle\int \csc\theta\,d\theta$

(c) $\displaystyle\int \frac{dx}{5x^2+9}$

(d) $\displaystyle\int \frac{dx}{1-2x}$

(e) $\displaystyle\int x^3\sqrt[3]{2x^4+5}\,dx$

(f) $\displaystyle\int \sec\theta\,\tan^3\theta\,d\theta$

(g) $\displaystyle\int \sin^5\theta\,\cos^5\theta\,d\theta$

(h) $\displaystyle\int \frac{x+1}{\sqrt{x-5}}\,dx$

(i) $\displaystyle\int \frac{e^x}{e^x+1}\,dx$

(j) $\displaystyle\int e^{x/3}\sin(2x)\,dx$

(k) $\displaystyle\int \frac{x^2-1}{x+1}\,dx$

(l) $\displaystyle\int \sqrt{7x-9}\,dx$

(m) $\displaystyle\int \frac{\sin(\ln x)}{x}\,dx$

(n) $\displaystyle\int \sin(2x)\tan x\,dx$

(o) $\displaystyle\int_1^9 \frac{\ln x}{\sqrt{x}}\,dx$

(p) $\displaystyle\int \csc^4\theta\,\cot^6\,d\theta$

(q) $\displaystyle\int_0^1 \sin^{-1}\theta\,d\theta$

Problem 3: Evaluate the following:

(a) $\displaystyle\int \sec\theta\,d\theta$

(b) $\displaystyle\int e^{\sin\theta}\cos\theta\,d\theta$

(c) $\displaystyle\int (2x+1)^{10}\,dx$

(d) $\displaystyle\int \frac{x^2+1}{x-1}\,dx$

(e) $\displaystyle\int x^2\sqrt{x-1}\,dx$

(f) $\displaystyle\int \csc^3\theta\,\cot^3\theta\,d\theta$

(g) $\displaystyle\int_{\pi/4}^{\pi/2} \cot^3\theta\,d\theta$

(h) $\displaystyle\int x^2\,2^x\,dx$

(i) $\displaystyle\int \frac{\ln x}{x}\,dx$

(j) $\displaystyle\int (x^2e^x+xe^x)\,dx$

(k) $\displaystyle\int \frac{dx}{6+5x^2}$

(l) $\displaystyle\int (4x+1)(2x^2+x)^8\,dx$

(m) $\displaystyle\int \frac{x^3}{(x^2+5)^2}\,dx$

(n) $\displaystyle\int x^3e^x\,dx$

(o) $\displaystyle\int \frac{x^3e^{x^2}}{(x^2+1)}\,dx$

(p) $\displaystyle\int \arctan\theta\,d\theta$

(q) $\displaystyle\int e^{2x}\sin(2x)\,dx$

(r) $\displaystyle\int \sin(3x)\cos(x)\,dx$

(s) $\displaystyle\int \frac{\tan^2\theta}{\sec^5\theta}\,d\theta$

(t) $\displaystyle\int \tan^6\theta\,d\theta$

Problem 4: Evaluate the following:

(a) $\displaystyle\int_1^{e^5} \frac{(\ln x)^6+3}{x}\,dx$

(b) $\displaystyle\int (\ln x)^2\,dx$

(c) $\displaystyle\int \frac{x-1}{x+1}\,dx$

(d) $\displaystyle\int (2x^2+4)\cos\left(\frac{x}{2}\right)\,dx$

(e) $\displaystyle\int_0^3 \frac{e^x-e^{-x}}{e^x+e^{-x}}\,dx$

(f) $\displaystyle\int \frac{2x-1}{3x-5}\,dx$

(g) $\displaystyle\int 4x^3\cos(3x)\,dx$

(h) $\displaystyle\int x^7\sqrt{2x^4+1}\,dx$

(i) $\displaystyle\int_1^{16} \frac{dx}{\sqrt{x}(1+\sqrt{x})^2}$

(j) $\displaystyle\int e^{2x}\cos x\,dx$

(k) $\displaystyle\int \sin^2(5\theta)\,d\theta$

(l) $\displaystyle\int e^{\pi x}\sin(\pi^2 x)\,dx$

(m) $\displaystyle\int \theta\tan^{-1}\theta\,d\theta$

(n) $\displaystyle\int \cos\theta\ln(\sin\theta)\,d\theta$

(o) $\displaystyle\int \frac{\tan^3\theta}{\sqrt{\sec\theta}}\,d\theta$

(p) $\displaystyle\int \frac{\cos\theta\,\sec\theta}{\csc\theta}\,d\theta$

(q) $\displaystyle\int 5x^2e^{x/5}\,dx$

Problem 5: Find the area between the given curves:

(a) $f(x)=x^2$, $g(x)=0$, $x=-2$, $x=2$

(b) $y=\sin x$, $y=\dfrac{4x}{\pi\sqrt{2}}$ in Quadrant 1.

(c) $f(x)=x^2-1$, $g(x)=1-x^2$

(d) $f(x)=x^2$, $y=4$, $x=0$

(e) $y = 1-(x-1)^2$, $x = \frac{1}{2}$, $y = 0$

(f) $f(x) = \sqrt[5]{x}$, $x = 0$, $y = 32$

(g) $y = x-1$, $y^2 = 2x+6$

(h) $x = y^2-4$, $x = y+2$

(i) $x = y^3-10y+3$, $x = 3-3y^2$

Problem 6: Find the average value of $f(x) = x^2+2x-1$ on $[0,4]$.

Problem 7: Consider the each of the following lines:

(i) x-axis

(ii) y-axis

(iii) $x = 7$

(iv) $x = -6$

(v) $y = 10$

(vi) $y = -5$

For each of the following, set-up *but do not integrate* an integral expression using *both* the Disk/Washer and Shells method to calculate to the volume resulting from revolving the region bound by the given curves around each of the lines above (do not set-up the integrals in the case where the given line passes through the region):

(a) $f(x) = \sqrt{x}$, $g(x) = 0$, $x = 1$

(b) $f(x) = x^2$, $g(x) = x$

(c) $y = 2x$, $y = 3x-1$

(d) $y = |x|$, $y = 2$

(e) $y = \sin x$, $y = 0$

(f) $f(x) = 1-x^2$, $g(x) = x^2-1$

(g) $y = 2x-4$, $x = 6$, $y = 0$

(h) $f(x) = \sqrt{x-1}$, $y = (x-1)^2$

(i) $y = 2x-1$, $y = 3x-1$, $x = 2$

(j) $x = 4-y^2$, $x = y^2-4$

Problem 8: Find the volume in each problem by using known cross-sections.

(i) The base of a solid is the region formed by $f(x) = x(x-1)$ and $y = 0$. The cross sections perpendicular to the x-axis are squares. Find the volume of the solid.

(ii) The base of a solid has boundary given by the curves $y = x^3$ and $y = x$. The cross sections perpendicular to the x-axis are semicircles. Find the volume of the solid.

(iii) The base of a solid has boundary given by the curves $f(x) = x^2 - 1$ and $g(x) = 1 - x^2$. The cross sections perpendicular to the x-axis are equilateral triangles. Find the volume of the solid. What would the integral be if the cross sections were semicircles?

(iv) Find the volume of a solid pyramid with square base that is 5 units tall and 20 units on the side.

(v) A regular cone has a base that is 4 units across and 5 units tall. Find the volume of the cone.

(vi) The base of a solid has boundary given by $y = 4 - x^2/9$ and $y = 0$. Cross sections perpendicular to the x-axis are $30° - 60° - 90°$ triangles with one leg in the plane. What is the volume of the solid? What if the hypotenuse were in the plane?

(vii) The base of a solid has boundary given by $y = \sqrt{4 - x^2}$ and $y = 0$. Cross sections parallel to the x-axis are rectangles with length in the plane and height twice the length. Find the volume of the solid.

(viii) The base of a solid has boundary given by the ellipse $4x^2 + 9y^2 = 9$. Cross sections perpendicular to the x-axis are isosceles right triangles with the hypotenuse lying in the plane. Find the volume of the solid.

(ix) The base of a solid has boundary given by $x^2 + y^2 = 4$. The cross sections perpendicular to the x-axis are equilateral triangles. Find the volume of the solid.

(x) The base of a solid is given by the curve $y = \sin x$ from 0 to π and the curve $y = 0$. Cross sections perpendicular to the x-axis are semicircles. Find the volume of the solid.

(xi) The base of a solid is given by the curves $y = \sqrt{x}$ and $y = x^2$. Slices perpendicular to the y-axis are rectangles with height a third the length of the side lying in the plane. Find the volume of the solid.

Recall:

(a) $A_{\text{square}} = s^2$

(b) $A_{\text{circle}} = \pi r^2$

(c) $A_{\text{triangle}} = \frac{1}{2} bh$

(d) $A_{\text{eq.-triangle}} = \frac{\sqrt{3}}{4} s^2$

(e) A $30° - 60° - 90°$ have sides in ratio $1 : \sqrt{3} : 2$

(f) A $45° - 45° - 90°$ have sides in ratio $1 : 1 : \sqrt{2}$