Problem 1: Determine whether the following series converge or diverge. Justify your answer completely.

(a) $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{2n-1}{3n+5}\right)$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{6^n}{4^n-1}$

(c) $\displaystyle\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{n+\ln n}{n^2+1}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{2^n n!}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+n}{n^4+3}$

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{e^n}$

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{n+\cos n}{n^3}$

(i) $\displaystyle\sum_{n=1}^{\infty} \arctan(n^2)$

(j) $\displaystyle\sum_{n=1}^{\infty} \frac{n+\sqrt{n}}{2n-1}$

(k) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

(l) $\displaystyle\sum_{n=2}^{\infty} \frac{n^2+n}{n^3-4}$

(m) $\displaystyle\sum_{n=1}^{\infty} (\arctan n)^n$

(n) $\displaystyle\sum_{n=1}^{\infty} \frac{2n-5}{\sqrt{3n^6-2}}$

(o) $\displaystyle\sum_{n=1}^{\infty} \sin\left(\frac{1}{\sqrt{n^7}}\right)$

(p) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{3n-1}{1-5n}\right)^n$

(q) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n^2}$

(r) $\displaystyle\sum_{n=1}^{\infty} \frac{5}{n^3+\sqrt{2n^6+3}}$

(s) $\displaystyle\sum_{n=1}^{\infty} \sin\left(\frac{1}{\sqrt[3]{n}}\right)$

(t) $\displaystyle\sum_{n=1}^{\infty} \frac{7}{3\sqrt[3]{n}-1}$

Problem 2: Determine whether the following series converge or diverge. Justify your answer completely.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-3}{2n^4+7}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{n5^n}{n!}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{(n^2+1)2^n}{n!\sqrt{n-1}}$

(d) $\displaystyle\sum_{n=1}^{\infty} n^4 \sin\left(\frac{1}{n^4}\right)$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^3}+7}{n^2+2n+3}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{2n-3}{\sqrt{3n^3-n+1}}$

(g) $\displaystyle\sum_{n=3}^{\infty} \frac{n^n}{n!}$

(h) $\displaystyle\sum_{n=2}^{\infty} \frac{n^2+n-1}{n^3+2n^2+1}$

(i) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n - 7}{6 + 5^n}$

(j) $\displaystyle\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{-n^2}$

(k) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3 + 2n - 1}{2n^3 - 5n + 7}$

(l) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{3n^2 + n - 1}{2n^2 + 5n - 4}\right)^n$

(m) $\displaystyle\sum_{n=4}^{\infty} \frac{2n^3 - n + 1}{n^4 + 2n - 1}$

(n) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n (2n)!}{n^n}$

(o) $\displaystyle\sum_{n=1}^{\infty} n^5 \sin\left(\frac{1}{n^3}\right)$

(p) $\displaystyle\sum_{n=0}^{\infty} n \left(\frac{2}{3}\right)^{n-1}$

(q) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{(n+2)^n}$

(r) $\displaystyle\sum_{n=6}^{\infty} \frac{2n + 3}{\sqrt{n^5 + 2}}$

(s) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{5^n (n^3 + 2)}$

(t) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 + 3n + 2}{3^n + 5}$

Problem 3: Determine whether the following series diverge, converge conditionally, or converge absolutely. Justify your answer completely.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{1 + \sqrt{n}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n} + \sqrt{n+1}}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{n^4 + 5}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2}{e^n}$

(e) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{\pi}{n}\right)$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

Problem 4: Determine whether the following series converge or diverge. If the series diverges, prove it. If the series converges, find the sum. Be sure to justify your answer completely.

(a) $\displaystyle\sum_{n=1}^{\infty} 5 \left(\frac{2}{3}\right)^n$

(b) $\displaystyle\sum_{n=0}^{\infty} (-3)^{2-n} 2^{n+1}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1 + 3^{n-1}}{5^n}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{5^{n-1}}{3^{2n-1}}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{15}{(-4)^{n+1}}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5} \left(\frac{2}{3}\right)^{2n-1}$

Problem 5: Determine the center, radius of convergence, and interval of convergence for the following power series:

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{x^{n+1}}{n2^n}$

(b) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^n}{\sqrt{n^2+6}}$

(c) $\displaystyle\sum_{n=0}^{\infty}\frac{(x+3)^n}{n!}$

(d) $\displaystyle\sum_{n=1}^{\infty}\frac{x^n}{n^4+5}$

(e) $\displaystyle\sum_{n=0}^{\infty} n!(x-1)^n$

(f) $\displaystyle\sum_{n=1}^{\infty} n^5x^n$

(g) $\displaystyle\sum_{n=2}^{\infty}\frac{x^n}{\ln n}$

(h) $\displaystyle\sum_{n=3}^{\infty}\frac{x^{2n+1}}{5n+7}$

Problem 6: Find the first 4 nonzero terms of the Taylor series for the given function at the given center.

(a) $f(x)=e^{-2x}$, $x=0$

(b) $f(x)=\cos(\theta)$, $\theta=7\pi$

(c) $f(x)=\dfrac{1}{x}$, $x=3$

(d) $f(x)=x^4-x+1$, $x=-1$

(e) $f(x)=\dfrac{1}{1-x}$, $x=1$

(f) $f(x)=\ln(x^2+1)$, $x=0$

Problem 7: Use Taylor series to find the following:

(a) $\displaystyle\int \cos x^2\,dx$

(b) $\displaystyle\int \sin x^2\,dx$

(c) $\displaystyle\int e^{-x^2}\,dx$

(d) $\displaystyle\sum_{n=1}^{\infty}\frac{n}{2^n}$

(e) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$

(f) $\displaystyle\lim_{x\to 0}\frac{\sin x}{x}$

(g) $\displaystyle\lim_{x\to 0}\frac{\ln(x+1)}{x}$

Problem 8: Use Taylor Series to approximate the following:

(a) $\ln(1.1)$

(b) $e^{0.1}$

(c) $\sqrt{3.8}$

(d) $\sin(0.1)$