Name: *Caleb McWhorter — Solutions* **MAT 296: HW 2**

Spring 2018 **Due: 01/31**

Problem 1: Integrate the following: $\displaystyle\int x^2 \ln x \, dx$

$\ln x$	$\frac{x^3}{3}$
$\frac{1}{x}$	x^2

$$\int x^2 \ln x \, dx = \frac{1}{3}x^3 \ln x - \int \frac{1}{x} \cdot \frac{x^3}{3} \, dx = \frac{1}{3}x^3 \ln x - \frac{1}{3} \int x^2 \, dx = \frac{1}{3}x^3 \ln x - \frac{x^3}{9} + C$$

Problem 2: Integrate the following: $\displaystyle\int \frac{3 \ln x}{x^4} \, dx$

$3 \ln x$	$-\frac{1}{3x^3}$
$\frac{3}{x}$	$\frac{1}{x^4}$

$$\int \frac{3 \ln x}{x^4} \, dx = -\frac{3 \ln x}{3x^3} - \int \frac{3}{x} \cdot \frac{-1}{3x^3} \, dx = -\frac{\ln x}{x^3} + \int \frac{dx}{x^4} = -\frac{\ln x}{x^3} - \frac{1}{3x^3} + C$$

Problem 3: Integrate the following: $\displaystyle\int \sqrt{x}\ln x\, dx$

$\ln x$	$\frac{2}{3}x^{3/2}$
$\frac{1}{x}$	$\sqrt{x}$

$$\int \sqrt{x}\ln x\, dx = \frac{2}{3}x^{3/2}\ln x - \int \frac{2x^{3/2}}{3x}\, dx = \frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\int \sqrt{x}\, dx = \frac{2}{3}x^{3/2}\ln x - \frac{4}{9}x^{3/2} + C$$

Problem 4: Integrate the following: $\displaystyle\int_0^1 x^3\sqrt{1-x^2}\, dx$

x^2	$\frac{(1-x^2)^{3/2}}{-3}$
$2x$	$x\sqrt{1-x^2}$

$$\begin{aligned}
\int_0^1 x^3\sqrt{1-x^2}\, dx &= -\frac{1}{3}x^2(1-x^2)^{3/2}\Big|_0^1 - \int_0^1 \frac{2x(1-x^2)^{3/2}}{-3}\, dx \\
&= (0-0) + \frac{2}{3}\int_0^1 x(1-x^2)^{3/2}\, dx \\
&= \frac{2}{3}\cdot -\frac{1}{5}(1-x^2)^{5/2}\Big|_0^1 \\
&= \frac{2}{3}\left[0-\left(-\frac{1}{5}\right)\right] \\
&= \frac{2}{15}
\end{aligned}$$

Problem 5: Integrate the following:

$$\int x^3 \cos(3x)\, dx$$

Solution.

u		dv
x^3	$+$	$\cos(3x)$
$3x^2$	$-$	$\frac{\sin(3x)}{3}$
$6x$	$+$	$\frac{-\cos(3x)}{9}$
6	$-$	$\frac{-\sin(3x)}{27}$
0		$\frac{\cos(3x)}{81}$

$$\int x^3 \cos(3x)\, dx = \frac{1}{3}x^3 \sin(3x) + \frac{3}{9}x^2 \cos(3x) - \frac{6}{27}x \sin(3x) - \frac{6}{81}\cos(3x) + C$$

$$= \frac{1}{3}x^3 \sin(3x) + \frac{1}{3}x^2 \cos(3x) - \frac{2}{9}x \sin(3x) - \frac{2}{27}\cos(3x) + C$$

Problem 6: Integrate the following:

$$\int e^{2x}\sin(3x)\,dx$$

u		dv
$\sin(3x)$	$+$	e^{2x}
$3\cos(3x)$	$-$	$\dfrac{e^{2x}}{2}$
$-9\sin(3x)$	$+$	$\dfrac{e^{2x}}{4}$

$$\int e^{2x}\sin(3x)\,dx = \frac{1}{2}e^{2x}\sin(3x) - \frac{3}{4}e^{2x}\cos(3x) - \int \frac{9}{4}e^{2x}\sin(3x)\,dx$$

$$\frac{13}{4}\int e^{2x}\sin(3x)\,dx = \frac{1}{2}e^{2x}\sin(3x) - \frac{3}{4}e^{2x}\cos(3x)$$

$$\int e^{2x}\sin(3x)\,dx = \frac{4}{13}\left(\frac{1}{2}e^{2x}\sin(3x) - \frac{3}{4}e^{2x}\cos(3x)\right) + C$$

$$\int e^{2x}\sin(3x)\,dx = \frac{2}{13}e^{2x}\sin(3x) - \frac{3}{13}e^{2x}\cos(3x) + C$$

$$\int e^{2x}\sin(3x)\,dx = \frac{2\,e^{2x}\sin(3x) - 3\,e^{2x}\cos(3x)}{13} + C$$