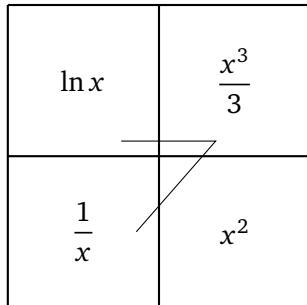
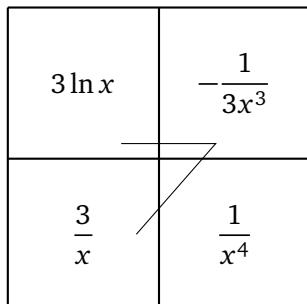


Problem 1: Integrate the following: $\int x^2 \ln x \, dx$



$$\int x^2 \ln x \, dx = \frac{1}{3}x^3 \ln x - \int \frac{1}{x} \cdot \frac{x^3}{3} \, dx = \frac{1}{3}x^3 \ln x - \frac{1}{3} \int x^2 \, dx = \frac{1}{3}x^3 \ln x - \frac{x^3}{9} + C$$

Problem 2: Integrate the following: $\int \frac{3 \ln x}{x^4} \, dx$



$$\int \frac{3 \ln x}{x^4} \, dx = -\frac{3 \ln x}{3x^3} - \int \frac{3}{x} \cdot \frac{-1}{3x^3} \, dx = -\frac{\ln x}{x^3} + \int \frac{dx}{x^4} = -\frac{\ln x}{x^3} - \frac{1}{3x^3} + C$$

Problem 3: Integrate the following: $\int \sqrt{x} \ln x \, dx$

| | |
|---------------|----------------------|
| $\ln x$ | $\frac{2}{3}x^{3/2}$ |
| $\frac{1}{x}$ | \sqrt{x} |

$$\int \sqrt{x} \ln x \, dx = \frac{2}{3}x^{3/2} \ln x - \int \frac{2x^{3/2}}{3x} \, dx = \frac{2}{3}x^{3/2} \ln x - \frac{2}{3} \int \sqrt{x} \, dx = \frac{2}{3}x^{3/2} \ln x - \frac{4}{9}x^{3/2} + C$$

Problem 4: Integrate the following: $\int_0^1 x^3 \sqrt{1-x^2} \, dx$

| | |
|-------|----------------------------|
| x^2 | $\frac{(1-x^2)^{3/2}}{-3}$ |
| $2x$ | $x\sqrt{1-x^2}$ |

$$\begin{aligned} \int_0^1 x^3 \sqrt{1-x^2} \, dx &= -\frac{1}{3}x^2(1-x^2)^{3/2} \Big|_0^1 - \int_0^1 \frac{2x(1-x^2)^{3/2}}{-3} \, dx \\ &= (0-0) + \frac{2}{3} \int_0^1 x(1-x^2)^{3/2} \, dx \\ &= \frac{2}{3} \cdot -\frac{1}{5}(1-x^2)^{5/2} \Big|_0^1 \\ &= \frac{2}{3} \left[0 - \left(-\frac{1}{5} \right) \right] \\ &= \frac{2}{15} \end{aligned}$$

Problem 5: Integrate the following:

$$\int x^3 \cos(3x) dx$$

Solution.

| <hr/> <i>u</i> | <i>dv</i> |
|----------------|------------------------|
| x^3 | $\cos(3x)$ |
| $3x^2$ | $\frac{\sin(3x)}{3}$ |
| $6x$ | $\frac{-\cos(3x)}{9}$ |
| 6 | $\frac{-\sin(3x)}{27}$ |
| 0 | $\frac{\cos(3x)}{81}$ |

$$\begin{aligned}\int x^3 \cos(3x) dx &= \frac{1}{3}x^3 \sin(3x) + \frac{3}{9}x^2 \cos(3x) - \frac{6}{27}x \sin(3x) - \frac{6}{81} \cos(3x) + C \\ &= \frac{1}{3}x^3 \sin(3x) + \frac{1}{3}x^2 \cos(3x) - \frac{2}{9}x \sin(3x) - \frac{2}{27} \cos(3x) + C\end{aligned}$$

Problem 6: Integrate the following:

$$\int e^{2x} \sin(3x) dx$$

| u | $d\nu$ |
|---------------|--------------------|
| $\sin(3x)$ | e^{2x} |
| $3 \cos(3x)$ | $\frac{e^{2x}}{2}$ |
| $-9 \sin(3x)$ | $\frac{e^{2x}}{4}$ |

$$\int e^{2x} \sin(3x) dx = \frac{1}{2} e^{2x} \sin(3x) - \frac{3}{4} e^{2x} \cos(3x) - \int \frac{9}{4} e^{2x} \sin(3x) dx$$

$$\frac{13}{4} \int e^{2x} \sin(3x) dx = \frac{1}{2} e^{2x} \sin(3x) - \frac{3}{4} e^{2x} \cos(3x)$$

$$\int e^{2x} \sin(3x) dx = \frac{4}{13} \left(\frac{1}{2} e^{2x} \sin(3x) - \frac{3}{4} e^{2x} \cos(3x) \right) + C$$

$$\int e^{2x} \sin(3x) dx = \frac{2}{13} e^{2x} \sin(3x) - \frac{3}{13} e^{2x} \cos(3x) + C$$

$$\int e^{2x} \sin(3x) dx = \frac{2 e^{2x} \sin(3x) - 3 e^{2x} \cos(3x)}{13} + C$$