**Name:** *Caleb McWhorter — Solutions* **MAT 296: HW 3**
**Spring 2018** **Due: 02/07**

**Problem 1:** Integrate the following: $\displaystyle\int x^2\sqrt{x-1}\,dx$

***Solution.*** *Let* $u = x-1 \iff x = u+1$. *Then* $du = dx$. *We have…*

$$\begin{aligned}\int x^2\sqrt{x-1}\,dx &= \int (u+1)^2\sqrt{u}\,du \\ &= \int (u^2+2u+1)\sqrt{u}\,du \\ &= \int (u^{5/2}+2u^{3/2}+u^{1/2})\,du \\ &= \frac{2}{7}u^{7/2}+\frac{4}{5}u^{5/2}+\frac{2}{3}u^{3/2}+C \\ &= \frac{2}{7}(x-1)^{7/2}+\frac{4}{5}(x-1)^{5/2}+\frac{2}{3}(x-1)^{3/2}+C\end{aligned}$$

**Problem 2:** Integrate the following: $\displaystyle\int \frac{x-1-\sqrt{x}+\sqrt[3]{x}}{\sqrt{x}}\,dx$

***Solution.***

$$\int \frac{x-1-\sqrt{x}+\sqrt[3]{x}}{\sqrt{x}} = \int \left(\sqrt{x}-\frac{1}{\sqrt{x}}-1+x^{-1/6}\right)dx = \frac{2}{3}x^{3/2}-2\sqrt{x}-x+\frac{6}{5}x^{5/6}+C$$

**Problem 3:** Integrate the following: $\displaystyle\int \frac{dx}{9x^2+4}$

$$\int \frac{dx}{9x^2+4} = \int \frac{dx}{9x^2+4} \cdot \frac{1/4}{1/4} = \frac{1}{4}\int \frac{dx}{\frac{9}{4}x^2+1} = \frac{1}{4}\int \frac{dx}{\left(\frac{3x}{2}\right)^2+1}$$

*Let* $u = \dfrac{3x}{2}$. *Then* $du = \dfrac{3}{2}\,dx \iff dx = \dfrac{2}{3}\,du$. *Thus, we have...*

$$\int \frac{dx}{9x^2+4} = \frac{1}{4}\int \frac{dx}{\left(\frac{3x}{2}\right)^2+1} = \frac{2}{3}\cdot\frac{1}{4}\int \frac{du}{u^2+1} = \frac{1}{6}\arctan(u) + C = \frac{1}{6}\arctan\left(\frac{3x}{2}\right) + C$$

**Problem 4:** Integrate the following: $\displaystyle\int x^3\sqrt[3]{2x^4+5}\,dx$

***Solution.*** *Let* $u = 2x^4+5$. *Then* $du = 8x^3\,dx$ *so that* $dx = \dfrac{du}{8x^3}$. *Then...*

$$\int x^3\sqrt[3]{2x^4+5}\,dx = \int \cancel{x^3}u^{1/3}\frac{du}{8\cancel{x^3}} = \frac{1}{8}\int u^{1/3}\,du = \frac{1}{8}\cdot\frac{3}{4}u^{4/3} + C = \frac{3}{32}(2x^4+5)^{4/3} + C$$

**Problem 5:** Integrate the following: $\int \sec\theta \tan^3\theta \, d\theta$

***Solution.***

$$\int \sec\theta \tan^3\theta \, d\theta = \int \tan^2\theta \cdot \sec\theta \tan\theta \, d\theta = \int (\sec^2\theta - 1) \cdot \sec\theta \tan\theta \, d\theta$$

*Let* $u = \sec\theta$. *Then* $du = \sec\theta \tan\theta \, d\theta$. *Therefore,*

$$\int \sec\theta \tan^3\theta \, d\theta = \int (\sec^2\theta - 1) \cdot \sec\theta \tan\theta \, d\theta = \int (u^2 - 1) \, du = \frac{u^3}{3} - u + C = \frac{\sec^3\theta}{3} - \sec\theta + C$$

**Problem 6:** Integrate the following: $\int e^{2x} \sin(2x) \, dx$

***Solution.***

| $u$ | sign | $dv$ |
|---|---|---|
| $\sin(2x)$ | $+$ | $e^{2x}$ |
| $2\cos(2x)$ | $-$ | $\frac{e^{2x}}{2}$ |
| $-4\sin(2x)$ | $+$ | $\frac{e^{2x}}{4}$ |

$$\int e^{2x} \sin(2x) \, dx = \frac{1}{2} e^{2x} \sin(2x) - \frac{2}{4} e^{2x} \cos(2x) - \int e^{2x} \sin(2x) \, dx$$

$$2 \int e^{2x} \sin(2x) \, dx = \frac{1}{2} e^{2x} \sin(2x) - \frac{1}{2} e^{2x} \cos(2x)$$

$$\int e^{2x} \sin(2x) \, dx = \frac{1}{4} e^{2x} \sin(2x) - \frac{1}{4} e^{2x} \cos(2x) + C$$

$$\int e^{2x} \sin(2x) \, dx = \frac{e^{2x} \sin(2x) - e^{2x} \cos(2x)}{4} + C$$

**Problem 7:** Integrate the following: $\displaystyle\int \cos\theta \, \ln(\sin\theta)\, d\theta$

***Solution.*** *Let* $u = \sin\theta$. *Then* $du = \cos\theta \, d\theta$. *Then*

$$\int \cos\theta \, \ln(\sin\theta)\, d\theta = \int \ln(\sin\theta)\cdot\cos\theta \, d\theta = \int \ln(u)\, du$$

| | |
|---|---|
| $\ln u$ | $u$ |
| $\frac{1}{u}$ | $1$ |

$$\int \ln(u)\, du = u\ln u - \int \frac{u}{u}\, du = u\ln u - \int 1\, du = u\ln u - u + C = \sin\theta\ln(\sin\theta) - \sin\theta + C$$

**Problem 8:** Integrate the following: $\displaystyle\int 4x^3\cos(3x)\, dx$

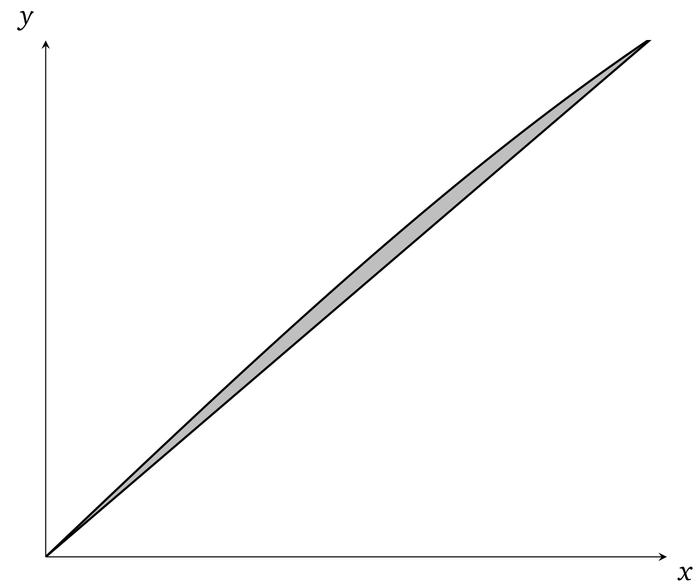
***Solution.***

| $u$ | | $dv$ |
|---|---|---|
| $4x^3$ | | $\cos(3x)$ |
| $12x^2$ | $+$ | $\frac{\sin(3x)}{3}$ |
| $24x$ | $-$ | $\frac{-\cos(3x)}{9}$ |
| $24$ | $+$ | $\frac{-\sin(3x)}{27}$ |
| $0$ | $-$ | $\frac{\cos(3x)}{81}$ |

$$\begin{aligned}\int 4x^3\cos(3x)\, dx &= \frac{4}{3}x^3\sin(3x) + \frac{12}{9}x^2\cos(3x) - \frac{24}{27}x\sin(3x) - \frac{24}{81}\cos(3x) + C \\ &= \frac{4}{3}x^3\sin(3x) + \frac{4}{3}x^2\cos(3x) - \frac{8}{9}x\sin(3x) - \frac{8}{27}\cos(3x) + C\end{aligned}$$

**Problem 9:** Find the area between $y = \sin x$, $y = \dfrac{4x}{\pi\sqrt{2}}$ in Quadrant I.

***Solution.*** *Clearly, if $x = 0$, we have $y = 0$ for both curves. Moreover, if $x = \dfrac{\pi}{4}$, we have $y = \dfrac{1}{\sqrt{2}}$ for both curves.*




*Then we have*

$$
\begin{aligned}
\textit{Area} &= \int_0^{\pi/4} \left( \sin x - \frac{4x}{\pi\sqrt{2}} \right) dx \\
&= \left[ -\cos x - \frac{4x^2}{2\pi\sqrt{2}} \right]_0^{\pi/4} \\
&= \left[ -\cos x - \frac{2x^2}{\pi\sqrt{2}} \right]_0^{\pi/4} \\
&= \left( -\cos(\pi/4) - \frac{2(\pi^2/16)}{\pi\sqrt{2}} \right) - (-\cos 0 - 0) \\
&= -\frac{1}{\sqrt{2}} - \frac{\pi}{8\sqrt{2}} + 1 \\
&= 1 - \frac{1}{\sqrt{2}} - \frac{\pi}{8\sqrt{2}}
\end{aligned}
$$