Name: *Caleb McWhorter — Solutions* **MAT 296: HW 4**
Spring 2018 **Due: 02/16**

Problem 1: What is the least common multiple of the following polynomials: $\{x-2, (x-2)(x-3)\}$?

Solution. *The least common multiple is* $(x-2)(x-3)$.

Problem 2: What is the least common multiple of the following polynomials: $\{x^2+x-6, x^2-7x+10\}$?

Solution. *Because* $\{x^2+x-6, x^2-7x+10\} = \{(x+3)(x-2), (x-2)(x-5)\}$, *the least common multiple is* $(x+3)(x-2)(x+5)$.

Problem 3: Solve the following system of equations:

$$\begin{aligned} a+b&=5 \\ a-b&=7 \end{aligned}$$

Solution. *Adding the equations gives* $2a = 12$ *so that* $a = 6$. *But then* $5 = a+b = 6+b$ *so that* $b = -1$.

Problem 4: Solve the following system of equations using elimination:

$$\begin{aligned} 2x-y+z&=8 \\ x-2y+z&=7 \\ -x+y-z&=-6 \end{aligned}$$

Solution. *Rewrite the first two equations, multiplying the second one by 2 gives*

$$\begin{aligned} 2x-y+z&=8 \\ 2x-4y+2z&=14 \end{aligned}$$

Subtracting them gives $3y-z=-6$. *Now rewrite the second equation and the third equation.*

$$\begin{aligned} x-2y+z&=7 \\ -x+y-z&=-6 \end{aligned}$$

Adding them gives $-y=1$ *so that* $y=-1$. *But we know* $3y-z=-6$, *so* $z=3y+6$. *Since* $y=-1$, *we have* $z=-3+6=3$. *Finally, we know* $x-2y+z=7$, *and using* $y=-1$ *and* $z=3$, *we have* $x+2+3=7$ *so that* $x=2$.

Problem 5: Integrate the following:

$$\int \frac{3\ln x}{x^4}\,dx$$

Solution.

$3\ln x$	$-\frac{1}{3x^3}$
$\frac{3}{x}$	$\frac{1}{x^4}$

$$\int \frac{3\ln x}{x^4}\,dx = -\frac{\ln x}{x^3} - \int \frac{3}{3x^4}\,dx = -\frac{\ln x}{x^3} - \frac{1}{3x^3} + C$$

Problem 6: Integrate the following:

$$\int \sin^3\theta\,d\theta$$

Solution.

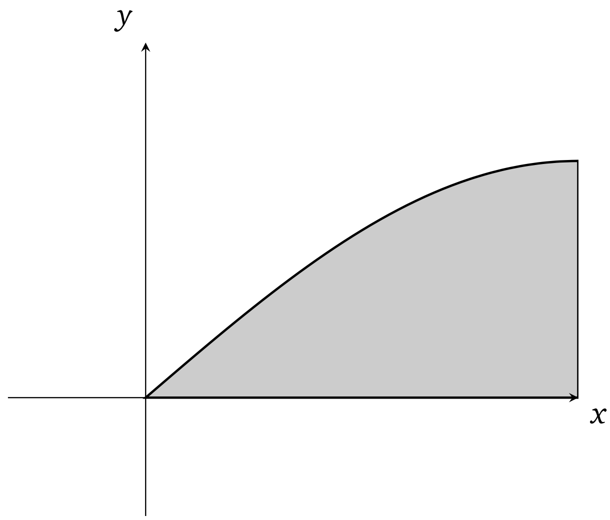
$$\int \sin^3\theta\,d\theta = \int \sin^2\theta\cdot\sin\theta\,d\theta = \int (1-\cos^2\theta)\cdot\sin\theta\,d\theta$$

Let $u = \cos\theta$. *Then* $du = -\sin\theta\,d\theta$.

$$\int \sin^3\theta\,d\theta = -\int (1-u^2)\,du = \int (u^2-1)\,du = \frac{u^3}{3} - u + C = \frac{\cos^3\theta}{3} - \cos\theta + C$$

Problem 7: Find the volume resulting from rotating the region bound by $y = \sin x$, $y = 0$, and $x = \dfrac{\pi}{2}$ about the y-axis.

Solution. The region is plotted below:



Using Shells, we have

$$V = \int_0^{\pi/2} x \sin x \, dx$$

x	$-\cos x$
1	$\sin x$

$$\begin{aligned} V &= 2\pi \int_0^{\pi/2} x \sin x \, dx \\ &= -2\pi x \cos x \Big|_0^{\pi/2} - 2\pi \int_0^{\pi/2} -\cos x \, dx \\ &= 2\pi \left(-x \cos x + \sin x\right)\Big|_0^{\pi/2} \\ &= 2\pi \left[\left(-\frac{\pi}{2} \cos\left(\frac{\pi}{2}\right) + \sin\left(\frac{\pi}{2}\right)\right) - (0+0)\right] \\ &= 2\pi \end{aligned}$$

<u>**OR**</u>

Using Disks/Washes, we have (since $y = \sin x \iff x = \sin^{-1}(y)$)

$$V = \pi \int_0^1 \left(\frac{\pi}{2}\right)^2 - \left(\sin^{-1} y\right)^2 \, dy$$

Now

$$\int_0^1 \left(\frac{\pi}{2}\right)^2 \, dy = \int_0^1 \frac{\pi^2}{4} \, dy = \frac{\pi^2}{4}$$

We only have to find

$$\int (\sin^{-1}(y))^2 \, dy = \int \arcsin^2 y \, dy$$

$\arcsin^2 y$	y
$\frac{2\arcsin(y)}{\sqrt{1-y^2}}$	1

$$\int \arcsin y \, dy = y \arcsin^2 y - \int \frac{2y \arcsin y}{\sqrt{1-y^2}} \, dy$$

$2\arcsin y$	$-\sqrt{1-y^2}$
$\frac{2}{\sqrt{1-y^2}}$	$\frac{y}{\sqrt{1-y^2}}$

$$\begin{aligned}\int \arcsin y \, dy &= y \arcsin^2 y - \int \frac{2y \arcsin y}{\sqrt{1-y^2}} \, dy \\ &= y \arcsin^2 y + 2\arcsin y \sqrt{1-y^2} - \int 2 \, dy \\ &= y \arcsin^2 y + 2\arcsin y \sqrt{1-y^2} - 2y + C\end{aligned}$$

Therefore, we have

$$\begin{aligned}\int_0^1 \arcsin^2 y \; dy &= \left(y \arcsin^2 y + 2 \arcsin y \sqrt{1-y^2} - 2y\right)\Big|_0^1 \\ &= \left(\arcsin^2(1) + 2\arcsin(1)\cdot 0 - 2\right) - (0+0-0) \\ &= \frac{\pi^2}{4} - 2\end{aligned}$$

Then finally, we have

$$\begin{aligned}V &= \pi \int_0^1 \left(\frac{\pi}{2}\right)^2 - \left(\sin^{-1} y\right)^2 \; dy \\ &= \pi \left[\int_0^1 \left(\frac{\pi}{2}\right)^2 \; dy - \int_0^1 \left(\sin^{-1} y\right)^2 \; dy\right] \\ &= \pi\left[\frac{\pi^2}{4} - \left(\frac{\pi^2}{4} - 2\right)\right] \\ &= 2\pi\end{aligned}$$