Name: *Caleb McWhorter — Solutions* **MAT 296: HW 7**
Spring 2018 **Due: 03/26**

Problem 1: For each of the following, determine if the Divergence Test applies. If the Divergence Test does not apply, explain why. If the Divergence Test does apply, use it to show that the series diverges.

(i) $\displaystyle\sum_{n=1}^{\infty} n^2 \sin\left(\frac{1}{n^2}\right)$

$$\lim_{n\to\infty} n^2 \sin\left(\frac{1}{n^2}\right) = \lim_{n\to\infty} \frac{\sin(1/n^2)}{1/n^2} = 1$$

Therefore, the series $\displaystyle\sum_{n=1}^{\infty} n^2 \sin\left(\frac{1}{n^2}\right)$ *diverges by the Divergence Test.*

(ii) $\displaystyle\sum_{n=0}^{\infty} \frac{n+1}{n^2+3}$

$$\lim_{n\to\infty} \frac{n+1}{n^2+3} = 0$$

Then the divergence test does not *apply.*

(iii) $\displaystyle\sum_{n=1}^{\infty} n \sin\left(\frac{1}{\sqrt{n}}\right)$

$$\lim_{n\to\infty} n \sin\left(\frac{1}{\sqrt{n}}\right) = \lim_{n\to\infty} \sqrt{n} \cdot \sqrt{n} \sin\left(\frac{1}{\sqrt{n}}\right) = \lim_{n\to\infty} \sqrt{n} \cdot \frac{\sin(1/\sqrt{n})}{1/\sqrt{n}} = \infty$$

Therefore, the series $\displaystyle\sum_{n=1}^{\infty} n \sin\left(\frac{1}{\sqrt{n}}\right)$ *diverges by the Divergence Test.*

(iv) $\displaystyle\sum_{n=0}^{\infty} \frac{2n-3}{5n+1}$

$$\lim_{n\to\infty} \frac{2n-3}{5n+1} = \frac{2}{5}$$

Therefore, the series $\displaystyle\sum_{n=0}^{\infty} \frac{2n-3}{5n+1}$ *diverges by the Divergence Test.*

(v) $\displaystyle\sum_{n=0}^{\infty} \cos\left(\frac{n-1}{n^2+3}\right)$

$$\lim_{n\to\infty} \cos\left(\frac{n-1}{n^2+3}\right) = \cos(0) = 1$$

Therefore, the series $\displaystyle\sum_{n=0}^{\infty} \cos\left(\frac{n-1}{n^2+3}\right)$ *diverges by the Divergence Test.*

(vi) $\displaystyle\sum_{n=1}^{\infty} \left(1+\frac{2}{5n}\right)^{3n}$

$$\lim_{n\to\infty} \left(1+\frac{2}{5n}\right)^{3n} = \lim_{n\to\infty} \left(1+\frac{1}{5n/2}\right)^{3n} = \lim_{n\to\infty} \left[\left(1+\frac{1}{5n/2}\right)^{5n/2}\right]^{\frac{2}{5}\cdot 3} = e^{6/5}$$

Therefore, the series $\displaystyle\sum_{n=1}^{\infty} \left(1+\frac{2}{5n}\right)^{3n}$ *diverges by the Divergence Test.*

(vii) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{(n+2)\sqrt{n}}$

$$\lim_{n\to\infty} \frac{n+1}{(n+2)\sqrt{n}} = \lim_{n\to\infty} \frac{n+1}{n^{3/2}+2\sqrt{n}} = 0$$

Therefore, the Divergence Test does not apply.

(viii) $\displaystyle\sum_{n=0}^{\infty} (-1)^n$

$$\lim_{n\to\infty} (-1)^n = DNE$$

Therefore, the series $\displaystyle\sum_{n=0}^{\infty} (-1)^n$ *diverges by the Divergence Test.*

(ix) $\displaystyle\sum_{n=0}^{\infty} \arctan(n)$

$$\lim_{n\to\infty} \arctan(n) = \frac{\pi}{2}$$

Therefore, the series $\displaystyle\sum_{n=0}^{\infty} \arctan(n)$ *diverges by the Divergence Test.*