Math 121: Exam 2
Summer – 2018
06/12/2018
90 Minutes

Name: *Caleb M^cWhorter — Solutions*

Write your name on the appropriate line on the exam cover sheet. This exam contains 8 pages (including this cover page) and 7 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
Total:	70	

1. (10 points) Suppose you are told A, B, C, and D are events in a probability distribution. You are told that C and D are independent and the following:

$$P(A) = 0.50 \qquad P(B) = 0.55 \qquad P(A \text{ and B}) = 0.30$$
$$P(C) = 0.28 \qquad P(D) = 0.15$$

(a) Could A and B be disjoint? Explain.

If A and B were disjoint, then $P(A \text{ and } B) = 0$. But then $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = P(A) + P(B) = 0.50 + 0.55 = 1.05 > 1$. Therefore, A and B cannot be disjoint. Alternatively, $P(A \text{ and } B) \neq 0$.

(b) Could A and B be independent? Explain.

If A and B were independent, then $P(A \text{ and } B) = P(A)P(B)$. But $0.30 = P(A \text{ and } B) \neq P(A)P(B) = 0.50 \cdot 0.55 = 0.275$. Therefore, A and B cannot be independent.

(c) Compute $P(A \text{ or } B)$.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 0.50 + 0.55 - 0.30 = 0.75$$

(d) Compute $P(C \text{ and } D)$.

Since C and D are independent,

$$P(C \text{ and } D) = P(C)P(D) = 0.28 \cdot 0.15 = 0.042$$

(e) Compute $P(\overline{B})$.

$$P(\overline{B}) = 1 - P(B) = 1 - 0.55 = 0.45$$

2. (10 points) A club consists of 18 members.

   (a) How many ways can the club choose a president, vice president, secretary, and treasurer from its members?

$${}_{18}P_4 = \frac{18!}{14!} = 73,440$$

   (b) How many ways can the club choose a committee of 3 members to plan a party for the club?

$${}_{18}C_3 = \frac{18!}{3!\,15!} = 816$$

Suppose you are told that the 18 members of the club consist of 10 women and 8 men.

   (c) How many ways can the club choose officers consisting of a female president and female treasurer but male vice president and male treasurer?

$${}_{10}P_2 \cdot {}_{8}P_2 = 90 \cdot 56 = 5,040$$

   (d) How many ways can the club form a committee of 5 people if the committee is made up of 3 men and two women?

$${}_{8}C_3 \cdot {}_{10}C_2 = 56 \cdot 45 = 2,520$$

3. (10 points) A game at a fair consists of drawing a random number from a bag filled with the numbers one through four, not necessarily the same amount of each. Once a number if drawn, it is replaced in the bag. If a 1 is drawn, you pay \$2. If a 2 is drawn, you pay \$1. If a 3 is draw, you win nothing. If a 4 is drawn, you win \$3. The probabilities of winning each amount, i.e. drawing the number associated with the amount, is summarized in the table below:

Amount	$-\$2$	$-\$1$	\$0	\$3
Probability	0.15	0.35	*0.30*	0.20

(a) Fill in the probability for \$0 above to make the table a probability distribution.

$$0.15 + 0.35 + 0.20 = 0.70$$

$$1 - 0.70 = 0.30$$

(b) What is the average amount a player expects to win per game in the long run?

$$\mu = \sum xP(x) = (-2)(0.15) + (-1)(0.35) + 0(0.30) + 3(0.20) = -\$0.05$$

(c) Compute the standard deviation, σ, for the probability distribution. Show your work.

x	$x - \mu$	$(x-\mu)^2$	$(x-\mu)^2 P(x)$
-2	-1.95	3.80	0.57
-1	-0.95	0.90	0.32
0	0.05	0.00	0.00
3	3.05	9.30	1.86
			Total: 2.75

Therefore, $\sigma^2 = 2.75$ *so that* $\sigma = \sqrt{2.75} = 1.66$.

4. (10 points) The following table summarizes the passengers on the Titanic by class and survival.

	First Class	Second Class	Third Class	Crew	Total
Survived	199	119	174	214	706
Died	130	166	536	685	1517
Total	329	285	710	899	2223

(a) What was the probability that a person aboard the Titanic survived?

$$\frac{706}{2223} = 0.318$$

(b) What is the probability that a person aboard the Titanic was in third class?

$$\frac{710}{2223} = 0.319$$

(c) What is the probability that a person was in second class or died during the sinking?

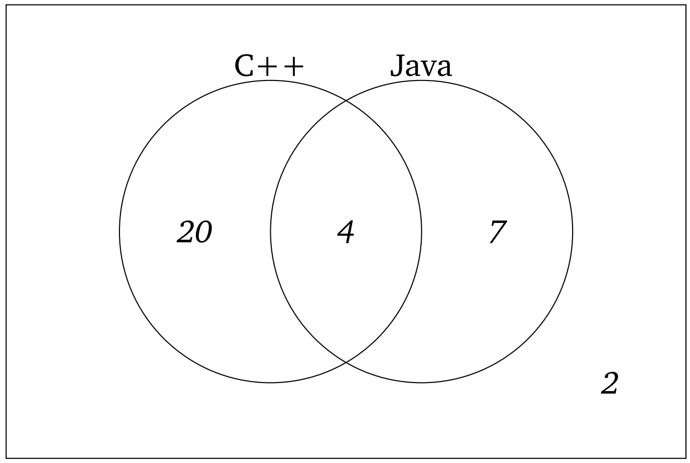
$$\frac{119 + 166 + 130 + 536 + 685}{2223} = \frac{1636}{2223} = 0.736$$

(d) What is the probability that a person was a crew member, given that they died during the sinking?

$$\frac{685}{130 + 166 + 536 + 685} = \frac{685}{1517} = 0.452$$

5. (10 points) Suppose in a programming class with 33 students, 24 students have seen C++, 11 students have seen Java, and 4 have seen both C++ and Java.

   (a) Complete the following Venn diagram by filling in the appropriate numbers:

   (b) What is the probability that a student has seen C++ but not Java?

   $$\frac{20}{33} = 0.606$$

   (c) What is the probability that a student has seen neither C++ nor Java?

   $$\frac{2}{33} = 0.06$$

   (d) What is the probability that a student has seen Java given that they have seen C++?

   $$\frac{4}{20+4} = \frac{4}{24} = 0.167$$

6. (10 points) A professor know that at her university, only 15% of students did not need to take out a loan to be able to attend the university. This professor is teaching a statistics course with 15 students.

   (a) Write a mathematical expression to compute the probability that *exactly five* of the students did not need to take out a loan to attend the university.

   $$P(N = 5) = {}_{15}C_5(0.15)^5(0.85)^{10} = 0.0449$$

   (b) Find the probability that *one or none* of the students did not need to take out a loan to attend the university.

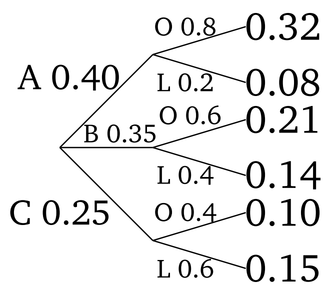
   $$P(N = 0) + P(N = 1) = 0.0874 + 0.2312 = 0.3186$$

   (c) Find the probability that *two or more* (including two) of the students did not need to take out a loan to attend the university.

   $$P(N \geq 2) = 1 - P(N < 2) = 1 - \big[P(N = 0) + P(N = 1)\big] = 1 - 0.3186 = 0.6814$$

7. (10 points) A particular city is serviced by three airlines for its passenger traffic. Airline A carries 40% of the passengers, Airline B carries 35%, and Airline C carries the remaining 25%. The probabilities that a passenger reaches his/her destination on time flying Airline A is 0.80, flying Airline B is 0.60, and flying airline C is 0.40.

   (a) Draw a tree diagram for a randomly selected airline passenger (which airline she takes and then whether she reaches her destination on time). Be sure to label the branches of the tree with appropriate probabilities and give the final probability for each branch.



   (b) What is the probability that a passenger will reach his/her destination on time?

$$P(\textit{on time}) = 0.32 + 0.21 + 0.10 = 0.63$$

   (c) If a passenger was late in reaching her destination, what is the probability that she flew Airline B?

$$P(B \mid \textit{late}) = \frac{0.14}{0.08 + 0.14 + 0.15} = \frac{0.14}{0.37} = 0.378$$