Math 121: Exam 3
Summer – 2018
06/21/2018
60 Minutes

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Write your name on the appropriate line on the exam cover sheet. This exam contains 8 pages (including this cover page) and 4 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	25	
2	25	
3	25	
4	25	
Total:	100	

1. (25 points) A cyber security company is hiring new analysts. The applicants are given an aptitude test as part of the interview process. The exam results were observed to be normally distributed with mean 2,032 (out of a possible 3,000 points) and standard deviation 374.

   (a) The company will automatically reject all applicants with scores below 1,946. What percent of applicants does the company automatically reject?

$$z_{1946} = \frac{1946 - 2032}{374} = -0.23 \rightsquigarrow 0.4090 = 40.90\%$$

   (b) The company will automatically hire all applicants with scores greater than 2,571. What percent of applicants does the company automatically hire?

$$z_{2571} = \frac{2571 - 2032}{374} = 1.44 \rightsquigarrow 0.9251$$

$$1 - 0.9251 = 0.0749 = 7.49\%$$

   (c) The company will subject applicants with scores between 1,946 and 2,571 for additional interviews to determine if they will be hired. What percent of applicants are subjected to additional interviews?

$$0.9251 - 0.4090 = 0.5161 = 51.61\%$$

(d) What is the minimum score would an applicant have to obtain to score in the top 12% of exam scores?

$$1 - 0.12 = 0.88$$

$$0.88 \rightsquigarrow z = 1.175$$

$$\begin{aligned} z &= \frac{x - \mu}{\sigma} \\ 1.175 &= \frac{x - 2032}{374} \\ 439.45 &= x - 2032 \\ x &= 2,471.45 \end{aligned}$$

An applicant would have to receive a score of at least 2,471.45.

(e) If 8 people are chosen at random, what is the probability that their average score will be less than 2,220?

$$z_{group} = \frac{2220 - 2032}{\frac{374}{\sqrt{8}}} = \frac{188}{132.229} = 1.42 \rightsquigarrow 0.9222$$

2. (25 points) A manufacturer of computer chips has received a large order for a new engineering firm. The firm has ordered 1,860 computer chips. Since the manufacturing process is not perfect, some defective chips will be produced. The failure rate of a chip for the manufacturing process is known to be 5.8%. To account for this, the manufacturer will produce 2,000 computer chips (140 chips more than ordered).

   (a) Find the average number of defective chips that will be produced.

$$\mu = np = 2000 \cdot 0.058 = 116$$

   (b) Find the standard deviation of the number of defective chips that will be produced.

$$\sigma = \sqrt{npq} = \sqrt{2000 \cdot 0.058 \cdot 0.942} = 10.45$$

   (c) Find the approximate probability that fewer than 140 chips will be defective.

$$z_{140} = \frac{140 - 116}{10.45} = 2.30 \rightsquigarrow 0.9893$$

(d) Use the continuity correction to improve your estimate in (c).

$$z_{139.5} = \frac{139.5 - 116}{10.45} = 2.25 \rightsquigarrow 0.9878$$

(e) Does it seem likely that the manufacturer will produce enough functional chips to fulfill the order?

The chance of them producing enough functioning chips is 98.78%. Therefore, it is likely that the company will be able to fulfill the order.

3. (25 points) A widget manufacturing company is constructing a new product. The company wishes to estimate the average production time to give good estimates for production and delivery times to their clients. Since the company has produced similar products in the past, they believe their procedure will result in a production time standard deviation of 3 hours. The company produces 46 components and finds it takes an average of 16.1 hours to construct a component.

   (a) Construct an 92% confidence interval for the average production time for this new component.

$$1 - 0.92 = 0.08$$
$$\frac{0.08}{2} = 0.04$$
$$0.92 + 0.04 = 0.96$$
$$0.96 \rightsquigarrow z = 1.75$$

$$\overline{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$
$$16.1 \pm 1.75 \cdot \frac{3}{\sqrt{46}}$$
$$16.1 \pm 0.77$$

$$(\ 15.33,\ 16.87\ )$$

   (b) Interpret the meaning to your answer in (a) in words.

We are 92% certain that the true mean production time is between 15.33 hours and 16.87 hours.

   (c) If the company wishes to construct a 95% confidence interval with error at most 0.25 hours for the mean production time, how large a sample size should be chosen?

$$n = \left\lceil \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2 \right\rceil$$
$$n = \left\lceil \left(\frac{1.96 \cdot 3}{0.25}\right)^2 \right\rceil$$
$$n = \left\lceil\ (23.52)^2\ \right\rceil$$
$$n = \lceil 553.19 \rceil$$
$$n = 554$$

4. (25 points) A drug manufacturing company is producing a new drug to help reduce the risk of hypertension. The company wishes to design a clinical drug trial to test the successfulness of the treatment. As part of the trial, the company will record whether a patient's blood pressure is reduced while taking the drug. In this way, the drug will be considered "effective" or "ineffective" in each patient.

   (a) Suppose the company wishes to construct a 99% confidence interval with error at most 5% for the drug's "effectiveness". How many people should be included in the drug trial to achieve this?

$$\begin{aligned} n &= \left\lceil \frac{z_{\alpha/2}^2 \cdot 0.25}{E^2} \right\rceil \\ n &= \left\lceil \frac{2.575^2 \cdot 0.25}{0.05^2} \right\rceil \\ n &= \left\lceil \frac{1.6577}{0.0025} \right\rceil \\ n &= \lceil 663.08 \rceil \\ n &= 664 \end{aligned}$$

   (b) Suppose a previous trial on a similar drug had been performed and the "effectiveness" rate was found to be 68%. Based on this information, if the company wishes to construct a 99% confidence interval with error at most 5% for the drug's "effectiveness", how many people should be included in the drug trial?

$$\begin{aligned} n &= \left\lceil \frac{z_{\alpha/2}^2 \cdot \hat{p}\,\hat{q}}{E^2} \right\rceil \\ n &= \left\lceil \frac{2.575^2 \cdot 0.68 \cdot 0.32}{0.05^2} \right\rceil \\ n &= \left\lceil \frac{1.443}{0.0025} \right\rceil \\ n &= \lceil 577.20 \rceil \\ n &= 578 \end{aligned}$$

(c) The company performs the study using 600 people. They find the drug is "effective" for 426 patients. Construct a 95% confidence interval for the "effectiveness" of this drug.

$$\frac{426}{600} = 0.71$$

$$\begin{aligned}
&\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\,\hat{q}}{n}} \\
&0.71 \pm 1.96\sqrt{\frac{0.71 \cdot 0.29}{600}} \\
&0.71 \pm 1.96 \cdot 0.0185 \\
&0.71 \pm 0.036
\end{aligned}$$

$$(\ 0.674,\ 0.746\ )$$

(d) Interpret the meaning to your answer in (c) in words.

We are 95% certain the the true average "effectiveness" of this drug is between 67.4% and 74.6%.