Math 121: Final Exam
Summer – 2018
06/28/2018
140 Minutes

Name: *Caleb M^{c}Whorter — Solutions*

Write your name on the appropriate line on the exam cover sheet. This exam contains 10 pages (including this cover page) and 9 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	5	
2	5	
3	10	
4	10	
5	10	
6	15	
7	15	
8	15	
9	15	
Total:	100	

1. (5 points) In each statement, identify which of these sampling methods is used: random, systematic, convenience, stratified, or cluster.

   (a) A student uses a Facebook poll to collect data for a statistics project.

   Convenience

   (b) A researcher polls 100 people from each of the 62 counties in New York.

   Stratified

   (c) An elementary school teacher draws three students' names out of a hat.

   Random

   (d) A pollster asks every sixth person leaving the polls for whom they cast their vote.

   Systematic

   (e) A political scientist randomly selects 10 law firms and interviews every lawyer at the firm.

   Cluster

2. (5 points) In each statement, determine which of the following level of measurements the underlined information represents: nominal, ordinal, interval, ratio

   (a) Alexander the Great was born in 356 B.C..

   Interval

   (b) She studies electrical engineering.

   Nominal

   (c) By land area, Australia is the sixth largest country in the world.

   Ordinal

   (d) There are 8.54 million people living in New York City.

   Ratio

   (e) The temperature outside is 32°C.

   Interval

3. (10 points) Consider the following dataset:

$$10,\ 10,\ 13,\ 15,\ 19,\ 23,\ 28,\ 28,\ 32,\ 33,\ 35,\ 39,\ 41,\ 46,\ 46,\ 61$$

(a) Draw a stem-and-leaf plot for this dataset.

1	0	0	3	5	9
2	3	8	8		
3	2	3	5	9	
4	1	6	6		
5					
6	1				

Stem Unit: 10

(b) Give the 5-number summary for this dataset.

Min	Q_1	Median	Q_3	Max
10	17	30	40	61

(c) Determine P_{60} for this dataset.

$$16 \cdot \frac{60}{100} = 9.6 \rightsquigarrow 10\textit{th number}$$

$$P_{60} = 33$$

4. (10 points) Complete each of the following parts.

(a) How many different ways can 8 people be arranged from left to right for a photo?

$$_8P_8 = 8! = 40,320$$

(b) How many different ways can you ask 3 people in a group of 6 friends to go out to grab food?

$$_6C_3 = \frac{6!}{3!(6-3)!} = 20$$

(c) In a club with 10 people, how many ways can the group elect a president, vice president, and activity director?

$$_{10}P_3 = \frac{10!}{(10-3)!} = 720$$

(d) How many distinct ways can the letters of the word 'statistics' be arranged?

$$\frac{10!}{3!3!2!} = 50,400$$

5. (10 points) The following table displays the number of professors that caught the flu, a cold, or neither (remainded healthy) this past winter.

	Flu	Cold	Healthy	Total
Male	4	18	65	87
Female	29	8	90	127
Total	33	26	155	214

Suppose a professor at this university is selected at random.

(a) What is the probability that the person was healthy all winter?

$$\frac{155}{214} = 0.7243$$

(b) What is the probability that the person was male?

$$\frac{87}{214} = 0.4065$$

(c) What is the probability that the person was female or had the flu?

$$\frac{29 + 8 + 90 + 4}{214} = \frac{127 + 33 - 29}{214} = \frac{131}{214} = 0.6121$$

(d) What was the probability that the person had a cold given that they were male?

$$\frac{18}{4 + 18 + 65} = \frac{18}{87} = 0.2069$$

6. (15 points) In a small community, only 35% of eligible voters actually voted in their local election. Suppose 14 eligible voters in this community are chosen at random.

   (a) What is the probability that exactly 4 of them voted in the election?

$$ {}_{14}C_4\ (0.35)^4(0.65)^{10} = 0.2022 $$

   (b) What is the probability that at most 3 of them voted in the election?

$$ P(N=0)+P(N=1)+P(N=2)+P(N=3) = 0.0024+0.0181+0.0634+0.1366 = 0.2205 $$

   (c) What is the probability that at least one of them voted in the election?

$$ 1 - P(N=0) = 1 - 0.0024 = 0.9976 $$

7. (15 points) The Mathematics Subject GRE has a distribution of scores which is normally distributed with mean 660 and standard deviation 140.

   (a) What is the probability that a student scores greater than 760 on the exam?

$$z_{760} = \frac{760 - 660}{140} = 0.71 \rightsquigarrow 0.7611$$

$$1 - 0.7611 = 0.2389$$

   (b) What is the minimum score required to be in the top 5% of test takers?

$$\begin{aligned} z &= 1.645 \\ z &= \frac{x - \mu}{\sigma} \\ 1.645 &= \frac{x - 660}{140} \\ 230.3 &= x - 660 \\ x &= 890.3 \end{aligned}$$

   (c) What is the probability that the average score of 7 students chosen at random is less than 600?

$$z = \frac{600 - 660}{\frac{140}{\sqrt{7}}} = \frac{-60}{52.9150} = -1.13 \rightsquigarrow 0.1292$$

8. (15 points) A news station wishes to estimate the proportion of Americans which support a new administration policy. They call 800 people at random and determine 344 people support the new policy.

   (a) Construct a 90% confidence interval to estimate the proportion of Americans who support the new policy.

$$\hat{p} = \frac{344}{800} = 0.43$$

$$\begin{aligned} \hat{p} &\pm z_{\alpha/2}\sqrt{\frac{\hat{p}\,\hat{q}}{n}} \\ 0.43 &\pm 1.645 \cdot \sqrt{\frac{0.43 \cdot 0.57}{800}} \\ 0.43 &\pm 0.0288 \end{aligned}$$

$$(\ 0.4012, 0.4588\ )$$

   (b) Assuming the news station had no prior estimate of the proportion of Americans that support the policy, how many people would the station have to survey to construct a 90% confidence interval for the proportion of Americans who support the new policy with an error at most 2%?

$$\begin{aligned} n &= \left\lceil \frac{z_{\alpha/2}^2 \cdot 0.25}{E^2} \right\rceil \\ n &= \left\lceil \frac{1.645^2 \cdot 0.25}{0.02^2} \right\rceil \\ n &= \left\lceil \frac{0.6765}{0.0004} \right\rceil \\ n &= \lceil 1691.25 \rceil \\ n &= 1,692 \end{aligned}$$

9. (15 points) In a survey of 18 people, a media company found participants spent an average of 113 minutes consuming various forms of media each day.

   (a) If the media company found a sample standard deviation of 39 minutes, construct a 95% confidence interval for the mean time spent consuming media each day.

$$\begin{aligned} \overline{x} &\pm t_{\alpha/2}\,\frac{s}{\sqrt{n}} \\ 113 &\pm 2.110\,\frac{39}{\sqrt{18}} \\ 113 &\pm 19.40 \end{aligned}$$

$$(\ 93.60,\ 132.40\ )$$

   (b) Construct a 95% confidence interval for the mean time spent consuming media each day if the population standard deviation was known to be 37 minutes.

$$\begin{aligned} \overline{x} &\pm z_{\alpha/2}\,\frac{\sigma}{\sqrt{n}} \\ 113 &\pm 1.96\,\frac{37}{\sqrt{18}} \\ 113 &\pm 17.09 \end{aligned}$$

$$(\ 95.91,\ 130.09\ )$$