Name: ________________________________ **MAT 121**

Summer 2018 **Exam 2 Review**

Problem 1: The table below classifies a group of voters according to gender and political affiliation:

	Democrat	Republican	Independent
Male	110	275	51
Female	301	138	58

(a) What is the total number of voters in this group?

(b) What percent of Republicans were male?

(c) Given that a voter was Independent, what is the probability that that the person was a male, assuming they were chosen at random?

(d) If a person is chosen at random from this group of voters, what is the probability of selecting a female or a Democrat voter?

Problem 2: A survey shows that 60% of university students own laptops. If 8 university students were selected at random, find...

(a) the probability that exactly 6 of them own laptops.

(b) the probability that at least 6 of them own laptops.

Problem 3: How many ways are there to select a committee of 3 members from among 10 faculty members?

Problem 4: A friend suggested the following game: A dice is rolled, if 6 is obtained, they will give you \$7, otherwise you must give him \$2. Compute your mean and standard deviation gain for this game.

Problem 5: How many ways can a Mathematics Department with 11 members choose a head, vice-head, secretary, and office coordinator — assuming no one person can hold two or more positions.

Problem 6: If $P(A) = 0.20$, $P(B) = 0.30$, and if A occurs then B has a probability of 0.32 of occurring, can A and B be independent? Explain.

Problem 7: The CEO of a car company claims that only 5% of used cars sold by the company have any manufacturing defects, aggravated by age, which may result in a crash. Assume the company selects 20 cars for inspection.

(a) Calculate the probability that less than 1 car has such a defect.

(b) Calculate the probability that one or less cars has such a defect.

(c) Calculate the probability that 2 or less of the cars have such a defect.

Problem 8: A die is rolled 3 times. What is the probability of getting at least one 4?

Problem 9: In an electoral race, any candidate from a certain political party has a 60% chance of winning an election. Suppose 8 candidates from this party are running for 8 positions in this election cycle.

(a) What is the probability that only one candidate from this party will win?

(b) What is the probability that at least one candidate from this party will win?

(c) What is the probability that this political party will win the majority of the available seats?

Problem 10: A majority of Americans favor the death penalty for convicted murder. A new Pew Research Center survey finds 53% of college graduates favor the death penalty, while 59% of those without a bachelor degree favor it. About 30% of U.S. adults graduated from college. Suppose an American adult is randomly selected. What is the probability that this person has a college degree if she/he favors the death penalty?

Problem 11: A club has 16 members. How many possible outcomes are there if it wishes to elect a finance committee with four members?

Problem 12: If $P(A) = 0.65$ and $P(B) = 0.45$, can A and B be disjoint? Explain your reasoning.

Problem 13: For a certain model of phones, the manufacturer offers a 3-year extended warranty for the price of $20. That means if the customer buys this extended warranty and the phone stops working within 3 years of the date of purchase, the manufacturer would replace the phone free-of-charge. Assume the probability of a new phone stops working within 3 years is 0.01 and it costs the manufacturer $150 to replace it, i.e. there is a 0.01 probability that the manufacturer will lose $130 for each warranty purchase.

(a) Find the probability distribution for the manufacturer's gain (X) per warranty purchase.

Manufacturer's Gain (X)	
Probability	

(b) In the long run, how much will the manufacturer gain per warranty purchase?

Problem 14: Consider a standard playing card deck of 52 cards, consisting of four suits of cards: spades, hearts, clubs, and diamonds. Each suit consists of the numbers 2–10, an ace, and three face cards: jack, queen, king. [Thus, each suit has 13 cards.] A typical poker hand consists of 5 cards. What is the probability of a player having a 'three of a kind', i.e. having three cards of the same type in their hand (assume they do not have four of the same card in their hand)?

Problem 15: In a class, there are 16 boys and 13 girls. If the teacher wants to select one boy and one girl for class representatives, how many ways can the teacher choose these representatives?

Problem 16: The following table summarizes the number of people of various ages and enjoyment of a recent movie:

	0–9	10–19	20–29	30 and over
Enjoyed	4	6	24	36
Not Enjoyed	8	12	45	65

(a) If a person is chosen at random, what is the probability that they enjoyed the movie?

(b) What proportion of people who watched the movie were 30 or older and did not enjoy the movie?

(c) Given that a person did not enjoy the movie, what is the probability that they were 20–29?

(d) Given that a person was 20–29, what is the probability that they did not enjoy the movie?

(e) Compare your answers in (b) and (c).

Problem 17: How many numbers are there between 99 and 1000 having a 7 in the units place? How many numbers in this range have at least one 7?

Problem 18: In a shipment of 100 apples, 70 are rotten, 40 are wormy, and 20 are both rotten and wormy. If one apple is chosen at random, what is the probability that it is either rotten or wormy?

Problem 19: The IRS audits 1% of companies in the US each years. The companies selected for audit in any one year can be assumed to be independent of the previous year's audit. [In reality, neither of these are true.]

(a) What is the probability that a company will be selected for audit twice over the next 8 years?

(b) What is the probability that a company will be selected three times over the next 15 years?

(c) What is the probability that a company will be selected at least once over the next 20 years?

Problem 20: A few years ago, the percentage of all employed people working for the U.S. Federal Government hit an all-time low, 2%. About 25% of U.S. workers are under 30 years old, while 0.14% are under 30 and work for the Federal Government.

(a) What percentage of U.S. Federal government employees are under 30 years old?

(b) If a U.S. Federal worker is randomly chosen, what is the probability that this person is under 30 or works for the Federal Government?

Problem 21: Consider drawing a card from a deck of 52 cards. Let A be the event that the card is an ace and let B be the event that the card is a spade. Are A and B independent events? Explain your reasoning mathematically.

Problem 22: A genetics expert has determined has determined that for certain couples, the probability of any child having a genetic disorder is 0.20.

(a) Consider a couple from this population who plans to have five children. Find the probability that at least one of their five children have a genetic disorder.

(b) Consider a couple from this population who plans to have five children. Use the binomial formula or table to compute the probability that exactly 2 of their 5 children have a genetic disorder.

Problem 23: How many different 5 letter 'words' can form from the letters of the word 'again'.

Problem 24: A shipment of 50 parts, including 5 that are defective is sent to an assembly plant. The plant quality control selects 3 parts at random (without replacement). What is the probability that *at least* one part selected by quality control is defective?

Problem 25: If $P(A) = 0.30$, $P(B) = 0.25$, and $P(A \text{ and } B) = 0.00$, can A and B be independent?

Problem 26: A club has 22 members.

(a) In how many ways can they choose 4 people to be on a committee?

(b) In how many ways can they choose a president, vice-president, and a treasurer?

Problem 27: A student takes a test with 8 multiple-choice questions, each with 5 possible answers. Because of a heavy social calendar, the student is unprepared and chooses the answer for each question by random selection. What is the probability that the student will get exactly 4 questions correct?

Problem 28: In a certain city, 95% of the households have cable TV. Suppose 15 households are chosen at random from that city.

(a) What is the probability that exactly 13 of the chosen households have cable TV?

(b) What is the probability that 13 or more of the chosen households have cable TV?

Problem 29: Suppose that 60% of students at a particular school are male, that 70% are in the regents diploma program, and that 45% are male *and* in the regents program. A student is to be chosen at random.

(a) Find the probability that the student will be male *or* in the regents program.

(b) Find the probability that the student will be neither a male nor in the regents program.

(c) Determine whether the events "the student is a male" and "the student is in the regents program" are *independent*.

Problem 30: Suppose that a certain unfair con has $P(\text{heads}) = 0.30$. This coin is to be flipped 9 times. Find...

(a) the expected number of times it will be heads.

(b) the probability that it will be heads exactly 3 times.

(c) the probability that it will be heads 2 or more times.

Problem 31: Of 20 white rabbits, 12 are female. Of 5 brown rabbits, 3 are female and 2 are male. if one of these 25 rabbits is chosen at random, what is the probability that it is either white or male?

Problem 32: A city is considering an amendment to its constitution. 55% of the voters in the city are conservatives and the remaining are liberals. 85% of the conservatives are in favor of the amendment. 35% of the liberals are also in favor of the amendment. Based on this, compute the following probabilities (to 4 decimal places).

(a) If a voter from that city is chosen at random, what is the probability that the voter chosen is a liberal and is against the amendment?

(b) If a voter from that city is chosen at random, what is the probability that the voter chosen is in favor of the amendment?

(c) If a voter from that city that is in favor of the amendment is chosen at random, what is the probability that the voter is a conservative?

Problem 33: A password is made according to the following rules: each password has 4 characters, the first two are letters of the alphabet, repeats allowed, and the last two are single digits numbers (0–9), repeats allowed. How many different passwords are possible? What if repeats were not allowed for either?

Problem 34: In a family of 6 children, what is the probability that there will be more boys than girls?

Problem 35: Ten people show up at a YMCA for a pick-up basketball game.

(a) In how many ways can the group be divided into two teams of 5 each?

(b) Suppose all ten lineup to practice free throws. How many different lineups are possible?

Problem 36: One the drive to the university, a professor travels through 11 stoplights. The probability of any one stoplight being red is 20%.

(a) Find the probability of having to stop least one red light on the way to the university.

(b) Find the probability of having to stop at no red lights on the way to the university.

Problem 37: The board of directors of a company has 8 members.

(a) How many different possibilities are there for a subcommittee consisting of 3 members?

(b) How many different possibilities are there for choosing a president, vice president, secretary, and treasurer assuming no person can hold more than one office?

Problem 38: A box contains 18 orange markers and 12 blue markers.

(a) Find the probability that a randomly selected marker is orange.

(b) Suppose 2 markers are randomly selected from the box, one after the other and without replacement. Find the probability that both markers are orange.

Problem 39: Suppose that A and B are independent events with probabilities $P(A) = 0.3$ and $P(B) = 0.7$. Find $P(A \text{ and } B)$ and $P(A \text{ or } B)$.

Problem 40: A large lot of modems contains 20% defective modems and 10 modems are randomly selected from this lot.

(a) Find the probability that exactly 3 of the 10 modems are defective.

(b) Find the probability that at least one of the 10 modems is defective.

Problem 41: There are 15 points in the plane, no three of which lie along a straight line, how many possible triangles can be drawn using any 3 points from this collection of points?

Problem 42: The following data summarizes the distribution of blood types (ABO type and Rh type) for 300 subjects randomly selected from a large population. Give your answers to three decimal places.

	0	A	B	AB
Rh positive	82	89	54	20
Rh negative	13	27	7	8

(a) If a subject is randomly selected from this population, find the probability of getting someone who has a type AB blood.

(b) If a donor is randomly selected from this population, find the probability of getting someone who has type AB as well as Rh negative blood.

(c) If a donor is randomly selected from this population, find the probability that they have Rh negative blood given that their blood type is AB.

(d) If a donor is randomly selected from this population, find the probability that they have AB blood given that their blood is Rh negative.

(e) If the probability that a person with type A or B blood is Rh negative.

(f) Let X be the event that a randomly selected subject has type AB blood and Y be the event that a randomly selected subject has Rh negative blood. Are X and Y disjoint? Are they independent? Provide mathematical justifications for your answers.

Problem 43: A signal is sent to ships out at sea by shining colored lights at the top of the lighthouse. There are 4 possible colors to shine. A signal can be sent by using one, two, three, or all four of the lights. How many signals can be sent? How many can be sent using two or more lights?

Problem 44: A corporation decides to give a drug test to its employees. Suppose 0.5% of its employees use drugs, i.e. the probability that a randomly chosen employee is a drug-user is 0.005. The drug test will correctly identify a non-user as testing negative 99% of the time.

(a) Draw a tree diagram to summarize the information.

(b) What is the probability that a randomly chosen employee will have a positive result?

(c) Suppose a randomly chosen employee is tested positive, what is the probability that the person is a non-user?

Problem 45: A genetics expert has determined that for certain couples, there is a 0.24 probability that any child will have an X-linked recessive disorder.

(a) What is the probability that a couple with 5 children have a single child with an X-linked recessive disorder?

(b) What is the probability that a couple with 5 children have more than one child with an X-linked recessive disorder?

Problem 46: A sports team needs to contain 4 men and 5 women. There are 20 men and 8 women to choose from. How many different teams can be formed?

Problem 47: In a certain swampy region, 5% of the mosquitoes carry the West Nile virus. Suppose 9 of the mosquitoes are chosen at random.

(a) What is the probability that exactly two of them carry the West Nile virus?

(b) What is the probability that less than two of them carry the West Nile virus?

Problem 48: Explain, without using mathematics, why ${}_{20}C_4 =_{20} C_{16}$.

Problem 49: Suppose $P(A) = 0.7$, $P(B) = 0.5$, and $P(A \mid B) = 0.9$. Find the following:

(a) $P(A \text{ and } B)$

(b) $P(A \text{ or } B)$

Problem 50: Is it true that ${}_nP_k = {}_nP_{n-k}$? Explain why or why not.

Problem 51: The Institute for Environmental Decisions performed a study involving people who purchased a Honda Civic and reported data on gender and whether or not the car purchased had a hybrid engine. Fifty-eight percent of the study participants were male and 42% were female. According to the sample data, 40% of the male and 30% of the female study participants bought a Civic with a hybrid engine. Suppose a study participant is randomly selected from the sample. What is the probability that she is female given that she bought a hybrid?

Problem 52: There are 3 identical die, each having 2 red faces, 2 green faces, and 2 blue faces.

(a) Find the missing entry in the table which gives the probability distribution of X, the number of *different* colors observed on the top faces when the dice are rolled.

x	1	2	3
$P(x)$	$\frac{1}{9}$		$\frac{2}{9}$

(b) Compute $E(X)$, the expected value of the number of different observed colors.

(c) Calculate the standard deviation for this distribution.

Problem 53: If you choose 6 different points on a circle, how many different lines can draw connecting these points?

Problem 54: Consider drawing a card from a standard playing deck of 52 cards. Let A be the event that the card is a 7, B be the event that the card is red, let C denote the even that the card is the King of Clubs, and let D denote the event that the card is a face card. Determine the following:

(a) $P(A)$

(b) $P(B)$

(c) $P(A \text{ or } B)$

(d) $P(A \text{ and } B)$

(e) Are A and B disjoint?

(f) Are C and D independent?

(g) Are B and C independent?

Problem 55: A lake is stocked with 300 rainbow trout and 700 lake trout. [There are no other fish in the lake besides the ones stocked.] Joe catches 6 fist on opening day and throws each one back after he catches it. Find the probability that Joe caught more rainbow trout than lake trout.

Problem 56: Steve sells cars at a local dealership. The number, X, of cars he sells on a given day is uncertain, but an estimate of the probability distribution for the number of cars he sells on a given day is

Cars Sold, X	0	1	2	3
Probability	0.20	0.45	0.25	0.10

(a) What is the average number of cars Steve expects to sell on a given day?

(b) Steve earns both a salary and a commission for selling cards so that his total earnings, Z, for the day are given by the following formula: $Z = 50 + 200X$. Find the mean of Z, which represents Steve's average daily earnings.

Problem 57: If the alphabet of a certain civilization contained 33 letters, how many possible 4 letter words could be formed if repetition was not allowed? What is repetition was allowed?

Problem 58: The following table summarizes the passengers on the Titanic by class and survival.

	First Class	Second Class	Third Class	Crew	Total
Survived	199	119	174	214	706
Died	130	166	536	685	1517
Total	329	285	710	899	2223

(a) What is the probability that a person aboard the Titanic perished?

(b) What is the probability that a person in First Class died?

(c) What is the probability that a person died on the Titanic, assuming they were Third Class?

(d) Of those that died or were a crew members, what percentage of those people survived?

Problem 59: In a large city, 45% of the population are registered Republicans and 55% are registered Democrats. Suppose three people are randomly selected.

(a) Find the probability that all three are Democrats.

(b) Find the probability that at least one of the three is a Democrat.

Problem 60: A bus route between points A and B has 4 possible routes. The bus route between B and C has 3 possible routes. If a student wants to travel from point A to point C and back, seeing as much as possible by not repeating any route, how many different bus routes could the student plan?

Problem 61: In a class there are 13 girls and 9 boys.

(a) How many ways can a committee of five people be chosen from three girls and two boys?

(b) What is the probability that a committee of five, chosen at random from the class, consists of three girls and two boys?

(c) How many of the possible committees of five have no boys?

(d) What is the probability that a committee of five, chosen at random from the class, consists only of girls?

Problem 62: A company finds that about 20% of all new employees resign during their first year of employment. Fifteen new employees are hired this January.

(a) Find the probability that exactly two of these employees will resign during their first year.

(b) Find the mean and standard deviation of the number of these employees that will resign during their first year of work.

Problem 63: National Public Radio's (NPR's) new stories are two minutes long, five minutes long, or six minutes long. Suppose 10% of the stories are two minutes long, 70% are five minutes long, and the rest are six minutes long. What is the mean length for news stories broadcasted by NPR?

Problem 64:

(a) Suppose 10 books are returned from editing (for errors). Let A be the event that all 10 are errorless. Describe in words the complementary event $\overline{A}$.

(b) In a certain town 70% of adults have health insurance. What is the probability that 6 randomly selected adults from this town all have health insurance?

(c) A couple plans to have 4 children. What is the probability that they will have at least one girl? [Assume that $P(\text{boy}) = P(\text{girl}) = \frac{1}{2}$.]

Problem 65: Your friend has 8 cards consisting of 4 kings and 4 queens which are randomly shuffled. You select without replacement 2 cards in sequence. Let A be the event that the first selection is a queen; let B be the event that the second selection is a king.

(a) Find the probability that both A and B occur.

(b) Find the probability that A or B occurs. [You may assume that $P(B) = \frac{1}{2}$.]

Problem 66: Many Dewitt telephone numbers have the form 446$-____$.

(a) How many such numbers are possible?

(b) How many are possible if the last four entries are all odd digits?

(c) How many are possible if the one of the remaining numbers must be a 4?

(d) How many are possible if all the remaining numbers must be different?

Problem 67:

(a) 10% of all Syracuse drivers were involved in a car accident last year. If 12 drivers are randomly selected, what is the probability of getting 2 or more who were involved in a car accident last year?

(b) Would it be unusual for exactly 3 of the 12 drivers to be involved in an accident?

Problem 68: The following table summarizes the flights from Syracuse, NY to NY, NY one weekend.

	On Time	Late
Podunk Air	33	6
Upstate Air	43	3

(a) Select one of the 85 flights at random. What is the probability that it was on time?

(b) Given that a flight was Upstate Air, what is the probability that it was on time?

Problem 69: How many four-digit *even* numbers can be formed using the digits 1, 2, 3, 4, 5, 6, 7, and 8?

Problem 70: An experiment consists of testing a substance 12 times for the presence of a contaminant. It is known that this occurs 10% of the time.

1. Find the expected number and the standard deviation for the number of times it occurs.
2. Find the probability that it occurs 2, 3, or 4 times.

Problem 71: A student has to answer 10 questions on a final exam. The final exam consists of two parts. Part A has 6 questions and Part B has 7 questions. If the student must answer at least 4 questions from Part A and at least 7 questions in Part B, how many different ways can the student choose the 10 questions?

Problem 72: A satellite has both a main and a backup solar power system. These two systems are independent by design. Suppose the probability of failure during a 10-year lifetime of the satellite is 0.1 for the main power system and 0.15 for the backup system.

(a) What is the probability that both systems will fail?

(b) Given that the main system has failed, what is the probability that the backup system will fail?

(c) What is the probability of the satellite having at least one functioning system?

(d) What is the probability that only the backup system is functioning?

Problem 73: Consider a standard playing card deck of 52 cards. For a game of poker, 5 cards are dealt to a player.

(a) Find the probability of a royal flush (10, jack, queen, king, and ace of a single suit in the hand).

(b) Find the probability of having four cards of the same type in your hand, aka a four of a kind.

Problem 74: A bag holds 12 red marbles, 4 yellow marbles, 8 blue marbles, and 16 white marbles.

(a) Assume that one marble is chosen from the bag. Find the probability that the marble chosen is yellow.

(b) Assume that three marbles are chosen from the bag with replacement. Find the probability that the first one is blue, the next is red, and the last one is white.

(c) Redo the previous part, assuming that the marbles are chosen without replacement.