Name: *Caleb McWhorter — Solutions* **MAT 121**
Summer 2018 **Homework 3**

Problem 1: Consider flipping a coin 6 times. Let A denote the set of events where at least 6 heads are obtained and B denote the set of events where at most 5 heads are obtained. Are A and B disjoint? What is $P(A$ or B$)$?

Solution. *Yes, A and B are disjoint; you cannot have 6 or more heads and 5 or less heads at the same time. Furthermore, A and B are complements! Since A and B are disjoint, $P(A \text{ and } B) = 0$. Then*

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = P(A) + P(B) = P(A) + P(\overline{A}) = 1.$$

Problem 2: Of 20 white rabbits, 12 are female. Of 5 brown rabbits, 3 are female and 2 are male. if one of these 25 rabbits is chosen at random, what is the probability that it is either white or male?

Solution. *A chart makes this simpler.*

	White	*Brown*
Male	*8*	*2*
Female	*12*	*3*

$$P(\text{white or male}) = \frac{8+12+3}{25} = \frac{23}{25} \approx 0.92$$

Problem 3: Physical features, specifically eye color and height, of students from a large lecture hall are observed. The data collected is summarized in the table below.

	Blue	Brown	Green
Short	42	32	1
Average	15	17	0
Tall	18	21	1

(a) What is the probability of a student chosen at random from the lecture is tall?

$$P(\text{tall}) = \frac{18+21+1}{147} = \frac{40}{147} \approx 0.272$$

(b) What is the probability of a student chosen at random from the lecture has blue eyes?

$$P(\text{blue eyes}) = \frac{42+15+18}{147} = \frac{75}{147} \approx 0.51$$

(c) What is the probability that a tall person from the lecture hall has brown eyes?

$$P(\text{brown eyes} \mid \text{tall}) = \frac{21}{18+21+1} = \frac{21}{40} = 0.525$$

(d) Given that a student has green eyes, what is the probability that the student is tall?

$$P(\textit{tall} \mid \textit{green eyes}) = \frac{1}{1+1} = \frac{1}{2} = 0.50$$

(e) Are the events "average height" and "green eyes" disjoint?

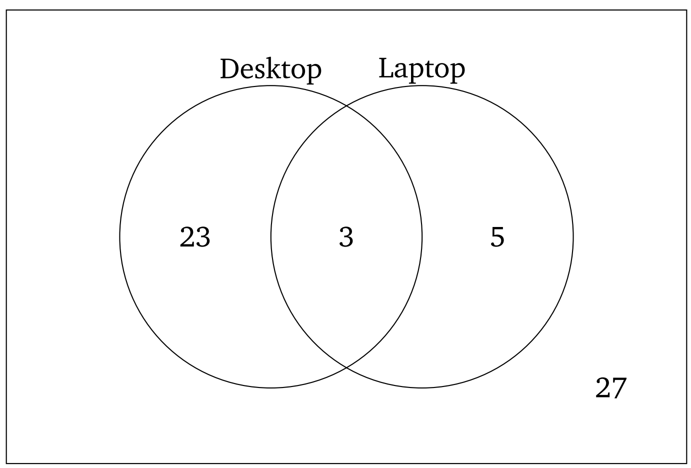
Yes, there are no students that have both green eyes and are of average height.

(f) Are the events "average height" and "green eyes" independent?

No, disjoint events are never independent; if a student has green eyes, then they cannot be average height and vice versa if a student is of average height they cannot have green eyes.

Problem 4: Suppose 58 people are interviewed about electronics they own. Of the people interviewed, 26 stated that the own a desktop, 8 people stated they own a laptop, and 3 stated that they own both.

(a) Draw a complete Venn diagram describing this scenario.



(b) What is the probability that a person chosen at random owns a desktop?

$$P(\textit{desktop}) = \frac{23+3}{58} = \frac{26}{58} = 0.448$$

(c) What is the probability that a person chosen at random owns neither?

$$P(\textit{neither}) = \frac{27}{58} \approx 0.466$$

(d) Choosing a person that owns some form of computer, what is the probability that they own a laptop?

$$P(\textit{laptop} \mid \textit{computer}) = \frac{3+5}{23+3+5} = \frac{8}{31} \approx 0.258$$

(e) Choosing. a person at random, what is the probability that they only own a desktop?

$$P(\textit{desktop only}) = \frac{23}{58} \approx 0.397$$

(f) Are the events "own a laptop" and "own a desktop" disjoint?

No, there are 3 people that own both a laptop and a desktop.

(g) Are the events "own a laptop" and "own a desktop" independent?

$$\begin{aligned} P(\textit{laptop}) &= \frac{3+5}{58} = \frac{8}{58} \approx 0.138 \\ P(\textit{desktop}) &= \frac{23+3}{58} = \frac{26}{58} \approx 0.448 \\ P(\textit{desktop and laptop}) &= \frac{3}{58} \approx 0.052 \\ P(\textit{laptop})P(\textit{desktop}) &= 0.138 \cdot 0.448 = 0.062 \end{aligned}$$

Because $P(\textit{desktop and laptop}) \neq P(\textit{laptop})P(\textit{desktop})$*, these events cannot be independent.*

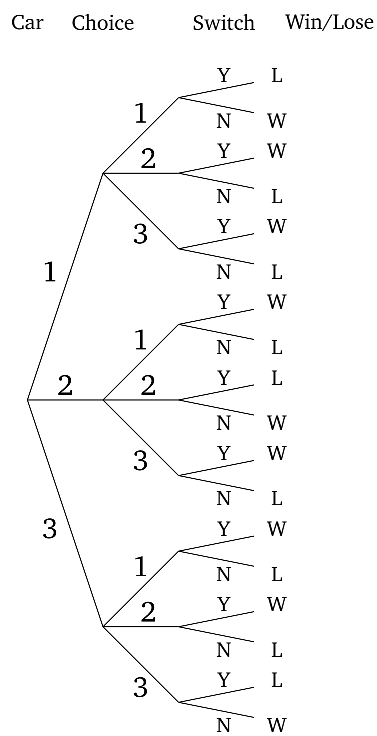
Problem 5: The Monty Hall problem is based off an old American television game show called *Let's Make a Deal* and is named after the shows original host, Monty Hall. The problem goes as follows:

You are on a game show. In front of you stand 3 doors. Behind one of the doors is a new car and behind the other two stand goats. You are to guess.a door. You will receive the contents of whatever is behind the door. You choose a door. Before opening the door, the host of the game show opens one of the doors you have not chosen which has a goat behind it. The host then offers you a choice: do you wish to switch your choice of door. Should you switch your door? Does it matter?

(a) Explain why a naïve person would believe that it does not matter whether one changes doors or not.

Since only two doors remain, one might think that the probability is 50-50. Hence, they would think switching would not affect the probability.

(b) Draw a complete event tree for this situation. [Note: in this case, we assume all the choices are equally likely, so your branches will have no probabilities, were are merely keeping track of all the possible outcomes.]



(c) Using (b), if one *does not* switch doors, what is the probability that one wins the car?

You do not *switch in 9 cases. Of the cases where you* do not switch, *you only win in 3 of these. Therefore, the probability is* $\frac{3}{9} = \frac{1}{3}$.

(d) Using (c), if one *does* switch doors, what is the probability that one wins the car?

You do *switch in 9 cases. Of the cases where you* do *switch, you win in 6 of these. Therefore, the probability is* $\frac{6}{9} = \frac{2}{3}$.

(e) Should the player switch doors when given the option?

Yes, you should always switch. The work above shows that this doubles your probability of winning the car. A reasoning for why this works is as follows: when first given the choice of door, you have a $\frac{1}{3}$ *chance of choosing the correct door, hence a* $\frac{2}{3}$ *chance of choosing the* incorrect *door. Meaning, you are unlikely to choose the door with the car. The host then eliminates an incorrect door. You can think of the host offering to let you switch doors as them allowing you to correct your likely first incorrect choice.*