Name: *Caleb McWhorter — Solutions* **MAT 121**
Summer 2018 **Homework 4**

Problem 1: If 7 people are running in a race, how many different first, second, and third place finishers are possible?

Solution.

$$_{7}P_{3} = \frac{7!}{4!} = 7 \cdot 6 \cdot 5 = 210$$

Problem 2: If you have 10 people available for a project, how many different groups of three people can you create from these ten people?

Solution.

$$_{10}C_{3} = \binom{10}{3} = \frac{10!}{3!7!} = 120$$

Problem 3: How many unique 'words' (meaning writing letters in order, i.e. from 'can' you can form the 'words': 'can', 'cna', 'nca', 'nac', 'anc', and 'acn') can you form from the word 'Mississippi'? How about the word 'achalasia'?

Solution.

$$\frac{11!}{4!\,4!\,2!} = 34,650$$

Problem 4: Consider the following chart describing a collection of numbers with associated probabilities:

x	-1	0	2	3	17	22.23
$P(x)$	0.18	0.06	0.24	0.33	0.16	0.03

(a) Does this table represent a probability distribution? Explain.

Yes, each value has an associated probability between 0 and 1 and the sum of the probabilities is 1:

$$0.18 + 0.06 + 0.24 + 0.33 + 0.16 + 0.03 = 1.0$$

(b) Find the expected value, i.e. the mean, for this table.

$$\mu = \sum xP(x) = -1(0.18) + 0(0.06) + 2(0.24) + 3(0.33) + 17(0.16) + 22.23(0.03) = 4.68$$

(c) Find the standard deviation for this dataset. *Therefore,* $\sigma^2 = 43.30$ *so that* $\sigma = \sqrt{43.30} = 6.58$.

x	$x - \mu$	$(x - \mu)^2$	$(x - \mu)^2 P(x)$
-1	-5.68	32.26	5.81
0	-4.68	21.9	1.31
2	-2.68	7.18	1.72
3	-1.68	2.82	0.93
17	12.32	151.78	24.29
22.23	17.55	308.0	9.24
			Total: 43.30

Problem 5: You are at a fair and there is a booth with a game. A wheel with the numbers 1 through 100 are on the wheel, each evenly spaced. If the spinner lands on 100, you win \$100, if the hand lands on the numbers 1–50, you have to pay \$2, and if the hand lands on 51–99, you win \$1. If you must pay \$0.50 to play the game each time you want to play, should you play this fair game?

Solution. *You win \$100 with probability* $\frac{1}{100}$, *pay \$2 with probability* $\frac{50}{100} = \frac{1}{2}$, *and win \$1 with probability* $\frac{49}{100}$. *On average, each game you win* $100(0.01) + (-2)(0.5) + 1(0.49) = \$0.49$. *But since you have to pay \$0.50 to play, this means on average you only make* $\$0.49 - \$0.50 = -\$0.01 = -1¢$. *Therefore, you should not play the game.*