Name: *Caleb McWhorter — Solutions* **MAT 121**

Summer 2018 **Homework 7**

Problem 1: Scores from a recent major college entry exam were approximately normal with mean 78 and standard deviation 5.

(a) What proportion of students scored below a 65?

$$z_{65} = \frac{65 - 78}{5} = -2.6 \rightsquigarrow 0.0047$$

(b) What proportion of students scored above a 85?

$$\begin{aligned} z_{85} &= \frac{85 - 78}{5} = 0.88 \rightsquigarrow 0.8106 \\ 1 - 0.8106 &= 0.1894 \end{aligned}$$

(c) What proportion of students scored between 65 and 85?

$$0.8106 - 0.0047 = 0.8059$$

(d) What score would a student need to receive in order to score in the top 7%?

$$\begin{aligned} 7\% \rightsquigarrow 93\% &\rightsquigarrow z_{0.93} \rightsquigarrow 1.48 \\ 1.48 &= \frac{x - 78}{5} \\ x &= 78 + 5(1.48) = 85.40 \end{aligned}$$

Problem 2: An Olympic archer named Kantmiss Evergreen is able to hit the bull's eye 90% of the time. Assume each shot is independent of the others.

(a) If she shoots 5 arrows, what is the probability that she misses the bull's eye exactly one?

$$_5C_1(0.10)^1(0.90)^4 \approx 0.3281 = 32.81\%$$

(b) If she shoots 5 arrows, what is the probability that the misses at most once?

$$P(X = 0) + P(X = 1) = 0.5905 + 0.3281 = 0.9186 = 91.86\%$$

(c) Suppose she will be shooting 200 arrows in a large competition. Let X be the number of bull's eyes she gets. What is the approximate distribution of the number of bull's eyes X? What are the mean and standard deviation of X?

The distribution is approximately normal:

$$\mu = np = 200(0.90) = 180$$
$$\sigma = \sqrt{npq} = \sqrt{200(0.90)(0.10)} = 4.24$$

(d) Suppose she made only 171 bully's eyes in 200 arrows. Use the normal approximation to estimate the probability that she makes 171 or less bull's eyes in 200 shots.

$$z_{171.5} = \frac{171 - 180}{4.24} = -2.12 \rightsquigarrow 0.0170 = 1.7\%$$

(e) Recalculate (d) using the continuity correction to improve the estimate.

$$z_{171.5} = \frac{171.5 - 180}{4.24} = -2.00 \rightsquigarrow 0.0228 = 2.28\%$$