

**Problem 1:** For a certain brand of light bulb, the mean lifetime is 6500 hours with a standard deviation of 400 hours. If a random sample of 52 of these light bulbs is chosen, what is the probability that the mean lifetime of the bulbs in that sample is at least 6450 hours?

$$z_{6450} = \frac{6450 - 6500}{\frac{400}{\sqrt{52}}} = \frac{-50}{55.47} = -0.90 \rightsquigarrow 0.1841$$

$$1 - 0.1841 = 0.8159$$

**Problem 2:** Based on a simple random sample, a 98% confidence interval for a proportion is given by (0.55, 0.65).

(a) Find the sample proportion  $\hat{p}$  and the margin of error  $E$ .

$$\hat{p} = \frac{0.55 + 0.65}{2} = 0.60$$

$$E = \frac{0.65 - 0.55}{2} = 0.05$$

(b) Find the necessary sample size  $n$  to obtain the above confidence interval.

$$z_{\alpha/2} = 2.33$$

$$n = \left\lceil \frac{z_{\alpha/2}^2 \hat{p} \hat{q}}{E^2} \right\rceil$$

$$n = \left\lceil \frac{2.33^2 \cdot 0.60 \cdot 0.40}{0.05^2} \right\rceil$$

$$n = \left\lceil \frac{1.3029}{0.0025} \right\rceil$$

$$n = \lceil 521.16 \rceil$$

$$n = 522$$

**Problem 3:** From a simple random sample of 420 students from a college, 189 of them said that they like their fitness center. Construct a 95% confidence interval for the proportion of all students who like the fitness center of such a college.

$$\hat{p} = \frac{189}{420} = 0.45$$

$$z_{\alpha/2} = 1.96$$

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.45 \pm 1.96 \sqrt{\frac{0.45 \cdot 0.55}{420}}$$

$$0.45 \pm 1.96 \cdot 0.0243$$

$$0.45 \pm 0.0476$$

$$( 0.4024, 0.4976 )$$

**Problem 4:** A simple random sample of 33 scores of statistics exams at a college has a sample average score of 80 points and a sample standard deviation of 8 points. Find a 99% confidence interval for the true population mean of all scores of statistics exams at such a college.

$$\bar{x} = 80$$

$$s = 8$$

$$n = 33$$

$$d.o.f. = 32$$

$$t_{\alpha/2} = 2.750$$

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

$$80 \pm 2.750 \cdot \frac{8}{\sqrt{33}}$$

$$80 \pm 3.83$$

$$( 76.17, 83.83 )$$