Name: *Caleb McWhorter — Solutions* **MAT 121**
Summer 2018 **Lab 2**

Problem 1: Let A and B denote events in a sample space with $P(A) = 0.4$, $P(B) = 0.7$, and $P(A \text{ and } B) = 0.2$.

(a) Are events A and B disjoint? Explain.

No, A and B cannot be disjoint because if so then $P(A \text{ or } B) = P(A) + P(B) = 0.4 + 0.7 = 1.1 > 1.0$, which is impossible.

(b) $P(A \text{ or } B) = P(A) + P(B) - P(A \textit{ and } B) = 0.4 + 0.7 - 0.2 = 0.9.$

(c) $P(\overline{A}) = 1 - P(A) = 1 - 0.4 = 0.6.$

Let events C and D denote independent events in a sample space with $P(C) = 0.30$ and $P(D) = 0.50$.

(d) $P(C \text{ and } D) = P(C)P(D) = 0.30 \cdot 0.50 = 0.15$

(e) $P(D \mid C) = \dfrac{P(C \textit{ and } D)}{P(C)} = \dfrac{0.15}{0.30} = 0.50$

Problem 2: The number of contracts, X, received by a consultant during a randomly selected month is according to the probability distribution is given below. It is also known that contracts received in different months are independent of each other.

Number Contracts, X	0	1	2	3	4
Probability	0.30	0.15	0.15	0.20	0.20

(a) Is this a probability distribution? Explain.

Yes, to each possible value is an associated probability, between 0 and 1, and the total of the probabilities is 1:

$$0.30 + 0.15 + 0.15 + 0.20 + 0.20 = 1.0$$

(b) Find μ, the mean of the probability distribution of X.

$$\mu = \sum xP(x) = 0(0.30) + 1(0.15) + 2(0.15) + 3(0.20) + 4(0.20) = 1.85$$

(c) Find σ^2, the variance of the probability distribution of X. Show all steps of the computation.

x	$x-\mu$	$(x-\mu)^2$	$(x-\mu)^2P(x)$
0	-1.85	3.42	1.03
1	-0.85	0.72	0.11
2	0.15	0.02	0.00
3	1.15	1.32	0.26
4	2.15	4.62	0.92
			Total: 2.33

Therefore, $\sigma^2 = 2.33$.

(d) What is the standard deviation for this distribution?

We have $\sigma^2 = 2.33$ *so that* $\sigma = \sqrt{2.33} = 1.53$.

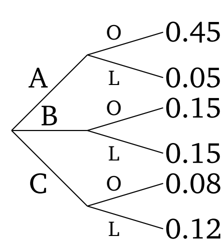
(e) If three months are selected at random, what is the probability that the consultant receives at least one contract in those three months?

This is the complement of the probability that the consultant receives no contracts in the three months:

$$1 - (0.30)(0.30)(0.30) = 1 - (0.30)^3 = 0.973$$

Problem 3: A particular city is serviced by three airlines for its passenger traffic. Airline A carries 50% of the passengers, Airline B carries 30%, and Airline C carries the remaining 20%. The probabilities that a passenger reaches his/her destination on time flying Airline A is 0.90, flying Airline B is 0.50, and flying airline C is 0.40.

(a) Draw a tree diagram for a randomly selected airline passenger (which airline she takes and then whether she reaches her destination on time) and label the branches of the tree with appropriate probabilities.



(b) What is the probability that a passenger will reach his/her destination on time?

$$P(\textit{on time}) = 0.45 + 0.15 + 0.08 = 0.68$$

(c) If a passenger was late in reaching her destination, what is the probability that she flew Airline C?

$$P(C \mid \textit{late}) = \frac{0.12}{0.05 + 0.15 + 0.12} = \frac{0.12}{0.32} = 0.375$$

Problem 4: If 9% of men are colorblind, if a SRS of 9 men is taken, what is the probability that at least one of the men is colorblind?

This is the complement that none of the men are colorblind:

$$P(\textit{none}) =_9 C_0 (0.09)^0 (0.91)^9 = 0.428$$
$$P(\textit{least one}) = 1 - P(\textit{none}) = 1 - 0.428 = 0.572$$

Problem 5: Consider rolling two die. Let A be the event that the first die lands on a 1. Let B denote the event that the second die shows a larger number than the first die. Let C denote the event that both die show the same number.

(a) Find $P(A)$, $P(B)$, and $P(C)$.

$$P(A) = \frac{1}{6} = 0.167$$
$$P(B) = \frac{15}{36} = 0.417$$
$$P(C) = \frac{6}{36} = 0.167$$

(b) Are A and B independent? Explain your reasoning.

$$P(A)P(B) = 0.167 \cdot 0.417 = 0.07$$
$$P(B \mid A) = \frac{1}{6} = 0.167$$
$$P(A \textit{ and } B) = P(A)P(B \mid A) = 0.167 \cdot 0.167 = 0.03$$

Since $P(A \textit{ and } B) \neq P(A)P(B)$, A and B are not independent.

(c) Are A and C independent? Explain your reasoning.

$$P(A)P(C) = 0.167 \cdot 0.167 = 0.03$$
$$P(C \mid A) = \frac{1}{6} = 0.167$$
$$P(A \textit{ and } C) = P(A)P(C)$$

This indicates that A and C are independent. Indeed, $P(C \mid A) = \frac{1}{6} = P(C)$ and $P(A \mid C) = \frac{1}{6} = P(A)$. Therefore, A and C are independent.

Problem 6: Consider a standard playing card deck of 52 cards, consisting of four suits of cards: spades, hearts, clubs, and diamonds. Each suit consists of the numbers 2–10, an ace, and three face cards: jack, queen, king. [Thus, each suit has 13 cards.] A typical poker hand consists of 5 cards.

(a) What is the total number of possible poker hands?

$${}_{52}C_5 = \binom{52}{5} = 2,586,960$$

(b) How many ways can a player have three and only three cards of the same type in their hand?

$${}_{13}C_1 \cdot {}_4C_3 = \binom{13}{1}\binom{4}{3} = 52$$

(c) How many ways can the other two cards be a 'two pair', meaning both be the same card?

$${}_{12}C_1 \cdot {}_4C_2 = \binom{12}{1}\binom{4}{2} = 72$$

(d) How many ways can a player have a 'full house', i.e. a three pair and a two pair in the same hand?

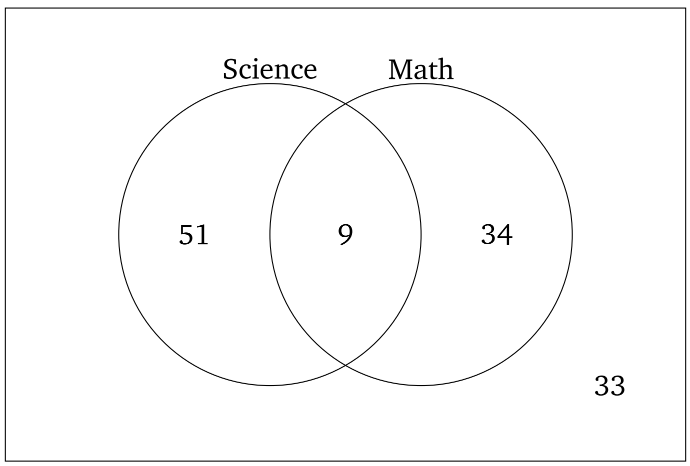
$${}_{13}C_1 \cdot {}_4C.3_{12}C_1 \cdot {}_4C_2 = \binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2} = 52 \cdot 72 = 3,744$$

(e) What is the probability of a player in poker having a full house?

$$\frac{{}_{13}C_1 \cdot {}_4C.3{}_{12}C_1 \cdot {}_4C_2}{{}_{52}C_5} = \frac{3,744}{2,586,960} = 0.001 = 0.1\%$$

Problem 7: Recent High School graduates and soon to be college Freshmen were surveyed to gauge their preparedness for college. Of 127 students surveyed, 60 had taken a Science class in the last year while 43 had taken a Math class within the last year. Interestingly of the students that took a Science class, 51 did not take a Math class. Assume a student from this survey is chosen at random.

(a) Draw a Venn diagram for this situation.



(b) What is the probability that the student took a Math class in the last year?

$$P(Math) = \frac{9+34}{127} = \frac{43}{127} = 0.339$$

(c) What is the probability that the student took a neither a Math nor a Science class in the last year?

$$P(neither) = \frac{33}{127} = 0.260$$

(d) What is the probability that the student only took a Math class in the last year?

$$P(\textit{only Math}) = \frac{34}{127} = 0.268$$

(e) What is the probability that the student took a Math class in the last year given that they had taken a Science class in the last year?

$$P(\textit{Math} \mid \textit{Science}) = \frac{9}{51+9} = \frac{9}{60} = 0.15$$

(f) What is the probability that a student either took only a Math class or neither a Math nor a science class?

$$P(\textit{Math or neither}) = \frac{9+34+33}{127} = \frac{76}{127} = 0.60$$

(g) Given that a student either took a Science class or neither class, what is the probability that they took a Science class?

$$P(\textit{Science} \mid \textit{Science or neither}) = \frac{51+9}{51+9+33} = \frac{60}{93} = 0.645$$

Problem 8: In a large city, 30% of people own smart phones. Suppose 12 people from the city are chosen at random.

(a) What is the probability that exactly 2 of the chosen people own a smart phone?

$$P(2) = {}_{12}C_2(0.30)^2(0.70)^{10} = 0.168$$

(b) What is the probability that less than 2 of the chosen people own a smart phone?

$$P(0) = {}_{12}C_0(0.30)^0(0.70)^{12} = 0.01P(1) = {}_{12}C_1(0.30)^1(0.70)^{11} = 0.07$$

$$P(\textit{less than } 2) = P(0) + P(1) = 0.01 + 0.07 = 0.08$$

(c) What is the probability that at least 3 of the chosen people own a smart phone?

This is the complement of the event where at most 2 own a smart phone. But then

$P(\textit{at least } 3) = 1 - P(\textit{at most } 2) = 1 - (P(2) + P(1) + P(0)) = 1 - (0.168 + 0.07 + 0.01) = 0.752$

Problem 9: From a group of 9 people, how many ways can two subcommittees be formed where one has four people and the other has three people?

$${}_9C_4 \cdot {}_5C_3 = 126 \cdot 10 = 1,260$$

Problem 10: A committee of two people is to be formed from a group of 30 people consisting of 10 men and 20 women.

(a) How many different possible committees are there?

$${}_{30}C_2 = \binom{30}{2} = 435$$

(b) How many of these committees are made up of two women?

$${}_{20}C_2 = \binom{20}{2} = 190$$

(c) If the committee is selected completely at random, what is the probability the committee is made up of two women.

$$\frac{190}{435} \approx 0.437$$