Name: *Caleb McWhorter — Solutions* **MAT 121**
Summer 2018 **Lab 3**

Problem 1: The mean replacement time for a random sample of 19 washing machines is 9.5 years and the standard deviation is 2.4 years.

(a) Construct a 90% confidence interval for the mean replacement time.

$$\begin{aligned}
&\overline{x} \pm z_{\alpha/2}\,\frac{\sigma}{\sqrt{n}} \\
&9.5 \pm 1.645 \cdot \frac{2.4}{\sqrt{19}} \\
&9.5 \pm 0.906 \\
&(\ 8.59,\ 10.41\ )
\end{aligned}$$

(b) Construct a 99% confidence interval for the standard deviation of the replacement time of all washing machines of this type.

$$\begin{aligned}
\frac{(n-1)s^2}{\chi^2_R} &< \sigma^2 < \frac{(n-1)s^2}{\chi^2_L} \\
\frac{18 \cdot 9.5^2}{37.156} &< \sigma^2 < \frac{18 \cdot 9.5^2}{6.256} \\
43.72. &< \sigma^2 < 259.671 \\
\sqrt{43.72} &< \sqrt{\sigma^2} < \sqrt{259.671} \\
6.61 &< \sigma < 16.11
\end{aligned}$$

Problem 2: The U.S. Marine Corps requires that male applicants have heights between 64 in and 78 in. Assume the heights of men are normally distributed with a mean of 68 in and standard deviation of 2.7 in.

(a) Find the percentage of men meeting those height requirements.

$$z_{78} = \frac{78-68}{2.7} = 3.70 \rightsquigarrow 1.0$$
$$z_{64} = \frac{64-68}{2.7} = -1.48 \rightsquigarrow 0.0694$$

$$1.0 - 0.0694 = 0.9306 = 93.06\%$$

(b) If the Secretary of Defense is to change the requirements so that only the shortest and tallest 3% of applicants are going to be denied, what are these heights?

$$z_{3\%} = -1.88$$
$$x = 68 + (-1.88) \cdot 2.7 = 62.92$$

$$z_{97\%} = 1.88$$
$$x = 68 + (1.88) \cdot 2.7 = 73.08$$

(c) If 32 men are randomly selected, find the probability that their mean height is greater than 67 in.

$$z_{67} = \frac{67-68}{\frac{2.7}{\sqrt{32}}} = \frac{-1}{0.4773} = -2.10 \rightsquigarrow 0.0179$$

$$1 - 0.0179 = 0.9821$$

Problem 3: The board of directors of a company has 8 members.

(a) How many different possibilities are there for choosing a president, vice president, secretary, and treasurer assuming no person can hold more than one office?

$$ {}_8P_4 = \frac{8!}{(8-4)!} = 1,680 $$

(b) How many different possibilities are there are for a subcommittee consisting of 3 members?

$$ {}_8C_3 = \frac{8!}{3!(8-3)!} = 56 $$

Problem 4: In a simple random sample pre-election poll of 1,400 voters, 800 say they intend to vote for candidate A. Construct a 99% confidence interval for the percentage of voters who intend to vote for candidate A.

$$ \hat{p} = \frac{800}{1400} = 0.5714 $$

$$ \hat{p} \pm z_{\alpha/2} \sqrt{\frac{0.5714 \cdot 0.4286}{1400}} $$

$$ 0.5714 \pm \sqrt{0.00017} $$

$$ 0.5714 \pm 0.0132 $$

$$ (\ 0.5582,\ 0.5846\ ) $$

Problem 5: A committee of two people is to be formed from a group of 30 people consisting of 10 men and 20 women.

(a) How many different possible committees are there?

$$_{30}C_2 = \frac{30!}{2!(30-2)!} = 435$$

(b) How many different committees are made up of one man and one woman?

$$_{10}C_1 \cdot \ _{20}C_1 = 10 \cdot 20 = 200$$

(c) How of these committees are made up of two women?

$$_{20}C_2 = \frac{20!}{2!(20-2)!} = 190$$

(d) If the committee is selected completely at random, what is the probability that the committee is made up of two women?

$$\frac{190}{435} = 0.4368$$