Math 222: Exam 1 **Name:** _____
Fall – 2019
10/03/2019
80 Minutes

Write your name on the appropriate line on the exam cover sheet. This exam contains 10 pages (including this cover page) and 5 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

1. uPhone advertises that their new '*XIV*' phone has download speeds of over 20 Mbps (megabytes per second). A consumer research agency suspects that the speeds are less than what is claimed in the advertisement. A random sample of download speeds using different times and locations was taken. The statistics are found below:

	n	$\overline{x}$	s
DLSpeed	30	18.4	3.7

(a) (6 points) Construct a 96% confidence interval for the mean download speeds for this phone. [You do *not need* to interpret the result.]

(b) (2 points) Write an appropriate null and alternative hypothesis for the given problem.

(c) (4 points) If the consumer research agency uses a significance level of 5%, what is the critical value for the test you wrote in (b)?

(d) (6 points) Given the sample data collected by the research agency, use the critical value in (c) to determine whether or not you reject the H_0 you gave in (b). State your conclusions in the context of the problem.

(e) (2 points) Given n, what must we assume about the sample for the previous parts to be accurate?

2. A particular survey course tends to be taken by either Freshmen or Seniors. The department is trying to determine if the course should only be available to those in their last two years. The department will make the decision based off course letter grades, only allowing students in their last two years to take the course if there is a statistically significant difference between Freshman and Senior grades. They take a random sample of students and find the following data:

	Total Number of Students	Number Receiving an 'A'
Freshman	40	10
Seniors	60	24

(a) (10 points) Using a significance level of 10%, conduct an appropriate hypothesis test to determine if the course should be limited to Juniors and Seniors. You must provide the null and alternative hypotheses, the test statistic, the p-value, and a conclusion stated in the context of the problem. Be sure to justify that the test is appropriate.

(b) (10 points) Find a 90% confidence interval for the difference in the *percentages* of Freshman and Seniors receiving an 'A' in the course. Interpret your answer in the context of the problem.

3. Education researchers are trying to determine if a course based on universal design (UD) principles increases students' performance in a course. They administer two independent Statistics courses, one based on UD principles (consisting of 45 students) and one not (consisting of 32 students). The UD course had a class average of 87.6 with standard deviation 3.3. The non-UD course had a class average of 82.3 with standard deviation 7.1.

   (a) (5 points) Is a pooled 2-sample t-test appropriate? Explain. Then regardless of whether a pooled test is appropriate, calculate s_p.

   (b) (5 points) Using an appropriate method, construct a 90% confidence interval for the difference in mean grade between the two courses. [You do not need to interpret the result.]

(c) (5 points) Use the confidence interval in (b) to test (at the 10% significance level) the null hypothesis $H_0 : \mu_{UD} = \mu_N$ against the alternative $H_a : \mu_{UD} \neq \mu_N$. State your conclusions in the context of the problem.

(d) (5 points) What if the researchers believed that students in a UD course should score 5 points higher on average than students that were not in a UD course. Is the data consistent with this hypothesis? Answer this by writing down an appropriate null and alternative hypotheses, calculating an appropriate test statistic and p-value, and state your conclusions at the 5% significance level in the context of the problem.

4. Red tide is a discoloration of seawater caused by blooms of a toxic red dinoflagellates (a type of plankton). When weather and tide factors cause these blooms, shellfish in the area develop dangerous levels of paralysis-inducing toxins. In Massachusetts, the Division of Marine Fisheries (DMF) tracks the levels of toxin in shellfish. If the mean level of toxin exceeds 800 μg of toxin per kilogram of clam meat, clam harvesting is banned. [The standard deviation of these toxins is known to be 51 μg.] The DMF uses a sample of 122 shellfish to test the following hypothesis:

$$\begin{cases} H_0 : \mu = 800 \\ H_a : \mu > 800 \end{cases}$$

The DMF decides to use a test that reject H_0 if $\overline{x} > 812.0\,\mu$g.

(a) (6 points) Find the probability of a Type I error for this test.

(b) (2 points) What is the significance level for this test?

(c) (2 points) What is the critical value for this test?

(d) (6 points) Is the test sufficiently sensitive to detect an increase of 5 μg of toxin per kilogram of clam meat? Answer this question by calculating the power of the test against the alternative $\mu = 805$ and interpreting the answer.

(e) (2 points) How can the DMF increase the power for this test to detect a change of 5 μg per kilogram of clam meat? Assume the DMF cannot change anything about the distribution of toxin levels, and that they do not want to change their significance level for the test.

(f) (2 points) Calculate the probability of a Type II error for this test if $\mu = 805$.

5. (20 points) Mark each of the following statements as True (T) or False (F).

(a) _____ t-procedures are robust.

(b) _____ The margin of error accounts for all possible sources of error in a confidence interval.

(c) _____ All other things equal, increasing the significance level will increase the probability of making a Type II error.

(d) _____ The larger (in absolute value) the test statistic is in a hypothesis test the more unusual the sample.

(e) _____ The power of a statistical test measures the ability to detect if the null hypothesis is false.

(f) _____ Failing to reject $H_0 : \mu = 52.1$ means that $\mu = 52.1$.

(g) _____ All other things equal, increasing the confidence level decreases the size of a confidence interval.

(h) _____ Two-sample t-tests are robust against the samples not being a SRS.

(i) _____ A Type I error is rejecting a true null hypothesis.

(j) _____ Assuming no sample bias and other sources of error, it is possible to compute the sample size required to estimate a population proportion p within a given margin of error with no prior information available about p.