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MAT 222
Fall 2019
Homework 2

"I'm scared that I'm not myself and I'm scared that I am"
–Piper Chapman, Orange is the New Black

Problem 1: Find the $z_{\alpha/2}$ value, i.e. the z^* value, for the confidence interval corresponding to the following α values:

(a) $\alpha = 0.20, \quad z_{\alpha/2} =$ 1.28

(b) $\alpha = 0.08, \quad z_{\alpha/2} =$ 1.75

(c) $\alpha = 0.02, \quad z_{\alpha/2} =$ 2.33

(d) $\alpha = 0.07, \quad z_{\alpha/2} =$ 1.81

Problem 2: What is the confidence level for each of the following confidence intervals for μ?

(a) Confidence Level: 95% ; $\overline{x} \pm 1.96 \cdot \dfrac{\sigma}{\sqrt{n}}$

(b) Confidence Level: 84% ; $\overline{x} \pm 1.41 \cdot \dfrac{\sigma}{\sqrt{n}}$

(c) Confidence Level: 78% ; $\overline{x} \pm 1.23 \cdot \dfrac{\sigma}{\sqrt{n}}$

Problem 3: Name 3 ways of decreasing the size of a confidence interval and a brief explanation of why your answer works:

- *Decreasing the confidence level: If you can be less confident about whether or not your interval contains the true mean, the interval can be smaller.*
- *Increasing the sample size: The larger the sample size, the more information you know about the population, so the confidence interval will be more accurate. Said differently, the larger the sample size, the smaller the standard deviation in the sampling distribution, so the margin of error is smaller, making the confidence interval smaller.*
- *Decreasing σ: The smaller σ is, the less variation there is so the more likely that your sample group values are close to the mean; hence, the sample mean will be close to the true mean. Said differently, the smaller the σ, the smaller the standard deviation in the sampling distribution, so the margin of error is smaller, making the confidence interval smaller.*

Problem 4: A random sample of 45 observations from a population produced the following summary statistics:

$$\sum x_i = 3654 \qquad \sum (x_i - \overline{x})^2 = 2475$$

(a) Assuming the sample standard deviation is approximately the true standard deviation, construct a 95% confidence interval for the population mean μ.

We know that $\overline{x} = \dfrac{\sum x_i}{n} = \dfrac{3654}{45} = 81.2$. *We know also* $s^2 = \dfrac{1}{n-1} \sum (x_i - \overline{x})^2$. *Then* $s^2 = \dfrac{1}{44} \cdot 2475 = 56.25$ *so that* $s = \sqrt{56.25} = 7.5$. *Assuming that* $s = \sigma$, *we know that* $\sigma = 7.5$. *Recalling that for a 95% confidence interval* $z^* = 1.960$, *we have*

$$\begin{array}{ccc} \overline{x} & \pm & z^* \dfrac{\sigma}{\sqrt{n}} \\ 81.2 & \pm & 1.960 \dfrac{7.5}{\sqrt{45}} \\ 81.2 & \pm & 2.19 \end{array}$$

which gives confidence interval $(79.01, 83.39)$.

(b) Interpret your answer from (a).

We are 95% sure that the true population mean μ *is between* 79.01 *and* 83.39.

Problem 5: A company in the US Southwest is secretive about what they pay their employees per hour. You go around to 39 different stores and ask 82 different employees their hourly pay. You find an average hourly pay of \$17.50/hr.

(a) Assuming the company's standard deviation in wages follows the normal standard deviation in store hourly pay of \$1.32/hr in the Southwest, construct a 90% confidence interval for their average employee hourly pay.

We have $n = 82$, $\overline{x} = 17.50$, *and* $\sigma = 1.32$. *For a 90% confidence interval, we have* $z_{\alpha/2} = 1.645$. *Then*

$$\overline{x} \quad \pm \quad z^* \frac{\sigma}{\sqrt{n}} = 17.50 \quad \pm \quad 1.645 \frac{1.32}{\sqrt{82}} = 17.50 \quad \pm \quad 0.24$$

so that we have a 90% confidence interval of $(17.26, 17.74)$.

(b) Assuming your confidence interval contains the population mean, what is the maximum possible error in your estimation of μ in (a)?

This is exactly the margin of error: $m = z^* \dfrac{\sigma}{\sqrt{n}} = \$0.24/$*hr.*

(c) How many employees would you have to randomly sample to estimate their hourly pay within \$0.16? [Assume the same level of confidence as the previous parts.]

To estimate the pay within ± 0.16*, we need* $m = 0.16$*. Then we have*

$$n \geq \left(\frac{1.645 \cdot 1.32}{0.16} \right)^2 = 184.179$$

so that we would need to survey at least 185 employees.

(d) In practice, does the method you used in (c) to find a minimal sample size to estimate a population mean μ with margin of error m sufficient?

No. This accounts only for random error and not necessarily other sources of error. Moreover, this would not take into account non-response or unusable responses.

Problem 6: Determine if the following statement is True or False: “The margin of error m takes into account all the error in estimating the population mean μ.”

The statement is false. The margin of error only takes into account error resulting from random sampling errors. The margin of error only expresses the error coming from random chance in the variation in randomized data production.