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MAT 222
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Homework 3

"I'm pretty, but tough like a diamond. Or beef jerky in a ball gown."
–Titus Andromedon, Unbreakable Kimmy Schmidt

Problem 1: A study on contaminated soil in the Netherlands looked at eighty-two 300g soil specimens, which were dried and analyzed for cyanide contamination. Contamination of cyanide (measured in mg/kg) in each soil sample was measured. The sample mean level of cyanide was found to be 85 mg/kg. It is known from previous samples of the soil that the standard deviation of cyanide contamination levels is 11.3 mg/kg.

If 5 years ago the soil contained an average cyanide level of 88 mg/kg, use a significance level of 5% to test the hypothesis that the contamination level has decreased over time. Be sure to state your null and alternative hypotheses, the critical value, test statistic with its p-value, and state the conclusions in the context of the problem.

Note that because $n = 82 \geq 30$, we can use a sampling distribution by the Central Limit Theorem. We have null and alternative hypothesis as follows,

$$\begin{cases} H_0 : \mu = 88 \\ H_a : \mu < 88 \end{cases}$$

Because $\alpha = 0.05$, to reject we need a 5% chance or less of seeing a value below the mean. This corresponds to a z-value of $z = -1.645$. Therefore, the critical value is -1.645. The test statistic is

$$z = \frac{85 - 88}{11.3/\sqrt{82}} = \frac{-3}{1.2478} = -2.40 \rightsquigarrow 0.0082$$

Therefore, the p-value is 0.0082. Because $p = 0.0082 < \alpha = 0.05$, we reject the null hypothesis. [We could also see this from the fact that the test statistic is smaller than the critical value.] Therefore, there is sufficient evidence to suggest that the mean level of cyanide contamination in this soil has decreased in the past 5 years.

Problem 2: As part of their training routine, a group of competitive cyclists spend time in a hyperbaric oxygen chamber to improve their competition performance. Given their previous training routines, the athletes had to spend an average of 8 hrs in the chamber each week (with standard deviation 3 hrs). However for an upcoming year of competition, the team needs to change their training routine. Sampling the athletes, they find an average time spent in the chamber to meet their required O_2 levels was approximately 6.9 hrs. This was based on a sample of size 37.

Construct a 99% confidence interval for the mean time needed to be spent in the chamber under this new training routine. Then use a significance level of 1% to test the hypothesis that there has been a change in the time required to spend in the chamber under this new training routine. Be sure to state your null and alternative hypotheses, the critical value, test statistic with its p-value, and state the conclusions in the context of the problem. Also, explain how you could have reached your decision using the confidence interval you constructed.

We have $n = 37$, $\overline{x} = 6.9$, and $\sigma = 3$. Notice $n = 37 \geq 30$ so that the Central Limit Theorem gives that a sampling distribution is appropriate. For a 99% confidence interval, we have $z^* = 2.576$. Then

$$\overline{x} \pm z^* \, \frac{\sigma}{\sqrt{n}} = 6.9 \pm 2.576 \, \frac{3}{\sqrt{37}} = 6.9 \pm 1.27$$

which gives confidence interval $(5.63, 8.17)$. Therefore, we are 99% certain that the true mean time required to be spend in the chamber is between 5.63 hours and 8.17 hours.

Now for the hypothesis test, we have null and alternative hypotheses

$$\begin{cases} H_0 : \mu = 8 \\ H_a : \mu \neq 8 \end{cases}$$

We reject if there is a 1% chance or less of seeing the given group mean. Because this test is two-sided, it can happen on either end so that we need a z-value corresponding to 0.5%. This gives critical value(s) -2.575 and 2.575. The test statistic is

$$z = \frac{6.9 - 8}{3/\sqrt{37}} = \frac{-1.1}{0.4932} = -2.23 \rightsquigarrow 0.0129$$

Therefore, the p-value is $2(0.0129) = 0.0258$. As $p = 0.0258 > \alpha = 0.01$, we fail to reject the null hypothesis. There is not sufficient evidence to suggest that the average time required to be spent in the chamber under the new training program has changed.

Note that as the test is two-sided with significance level $\alpha = 0.01$. We constructed the corresponding 99% confidence interval. Because 8 is in the interval $(5.63, 8.17)$, the data is consistent with the null hypothesis so that we would fail to reject the null hypothesis.

Problem 3: An outside management firm was hired to determine if there is a disparity in the pay between women of color and the average employee within a company. The overall pay in the company is approximately distributed as $N(37.3, 5.8)$, where the pay is measured in thousands of USD. A sample of 12 women of color was taken and they were found to have an average pay of \$33,000. Use a significance level of 1% to test the hypothesis that women of color at this company earn less than the average employee. Be sure to state your null and alternative hypotheses, the critical value, test statistic with its p-value, and state the conclusions in the context of the problem. Would you make the same conclusion if $\alpha = 0.05$? What about $\alpha = 0.001$? Explain.

Note that $n = 12$ is too normally too small to use the sampling distribution. However, we are told that the original distribution was approximately normal. Therefore, the sampling distribution is appropriate. We have $\overline{x} = 33$, $\sigma = 5.8$, and $n = 12$. We have null and alternative hypotheses

$$\begin{cases} H_0 : \mu = 37.3 \\ H_a : \mu < 37.3 \end{cases}$$

We reject if there is a 1% chance or less of seeing this group mean. This corresponds to a z-value of $z = -2.33$. We have a test statistic of

$$z = \frac{33 - 37.3}{5.8/\sqrt{12}} = \frac{-4.3}{1.6743} = -2.57 \rightsquigarrow 0.0051$$

This gives a p-value of 0.0051. Because $p = 0.0051 < \alpha = 0.01$, we reject the null hypothesis. There is sufficient evidence to suggest that women of color at this company earn less than the average employee. Notice that $p < \alpha$ if $\alpha = 0.05$ but $p > \alpha$ if $\alpha = 0.001$. Therefore, we would still reject the null hypothesis if $\alpha = 0.05$ but we would fail to reject the null hypothesis if $\alpha = 0.001$.