

Name: _____

MAT 222

Fall 2019

Homework 8

“[Miranda] You have no style or sense of fashion. [Andy] I think that depends on— [Miranda] No, no, that wasn’t a question.”

–Miranda Priestly & Andy Sachs, Devil Wears Prada

Problem 1: Education researchers are interested in studying equity in the SAT and ACT in college acceptances. The SAT and ACT are both supposedly exams to test students mastery of High School material and thus gauge their college readiness. Because both these exams test similar material, researchers are wondering how strong the relationship is between them. Specifically, researchers are testing if there is a predictive linear relationship between the two by testing if one can use a student’s SAT score to predict their ACT score. To test this hypothesis, the group takes a SRS of 60 students who took both the SAT and ACT. The data is summarized below:

$$\begin{array}{lll} \bar{x} = 912.667 & s_x = 180.112 & \sum(x_i - \bar{x})^2 = 1913973 \\ \bar{y} = 21.133 & s_y = 4.714 & \sum(y_i - \bar{y})^2 = 1310.93 \\ r = 0.8167 & s = 2.74353 & \\ b_0 = 1.63 & b_1 = 0.02137 & \end{array}$$

(a) For this sample, what was the average SAT score? What was the average ACT score?

(b) What was the resulting linear model for their statistical analyses?

(d) What is the standard error for b_0 ?

(e) Create a 99% confidence interval for the coefficient b_1 and interpret the result.

(f) What is the value of the coefficient of determination for this model? What does it tell you?

(g) What is the value of the MSE for this model?

(h) What is the value of SSE for this model?

(i) What is the value of SST?

(j) What is the value of SSM?

(k) Use SE_{b_1} to confirm that $\sum(x_i - \bar{x})^2 = 1913973$.

(l) Create a 90% confidence interval for the ACT score of a student who receives an SAT score of 1100. Interpret the result.

(m) What is the *mean* ACT score for a student who receives an SAT score of 1100?

(n) If the sample consisted of students who only received between 500 and 1200, should one use the model to predict the ACT scores for students whom received an SAT score of 1600? Explain.

Problem 2: College loan debt is a major drain on the US national economy. In an effort to try to predict future collection student loan debt, a government research group is attempting to predict the average amount of debt students exiting from different universities will have. They begin by analyzing debt from public universities. Using data from Kiplinger’s “Best Values in Public Colleges”, they try to predict average debt based on admittance rate, graduation rate, total in-state cost & aid, and total out-of-state cost & aid using data from 25 different schools. All currency was measured in tens of thousands of dollars. A summary of the model data is found below.¹

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	___	2.38156	_____	2.43	0.073
Admit	___	1.29861	1.29861	6.62	0.019
GradRat	___	_____	0.29454	1.50	0.235
TCostInS	___	0.28563	0.28563	_____	0.242
InCostAid	___	0.00085	0.00085	0.00	0.948
TCostOut	___	0.33604	0.33604	1.71	0.206
Error	___	_____	_____		
Total	___	6.10974			

Model Summary

S	R-sq	R-sq (adj)	R-sq (pred)
_____	_____%	22.92%	0.00%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	0.6819	_____	0.94	0.361	
Admit	1.803	0.701	2.57	0.019	1.20
GradRate	_____	0.755	1.23	0.235	1.67
TCostInS	0.585	0.484	_____	0.242	4.33
InCostAid	-0.027	0.410	-0.07	0.948	1.90
TCostOut	-0.275	0.210	-1.31	0.206	2.79

The regression equation is

$$\text{AvgDebt} = 0.6819 + 1.803 \text{ Admit} + 0.925 \text{ GradRat} + 0.585 \text{ TCostInS} - 0.027 \text{ InCostAid} - 0.275 \text{ TCostOut}$$

¹Note because there is a perfect linear relationship between out-of-state aid and the other variables, Minitab removes it from the model.

- (a) Fill in the missing entries in the table.
- (b) What does the model say the average loan debt is for a student that went to a public school with a admittance rate of 47 percent, graduation rate of 67 percent, total in-state cost of \$19,000, in-state aid of \$15,000, total out-of-state cost of \$31,500, and total out-of-state aid of \$24,000?
- (c) What is the coefficient of determination for this model?
- (d) Construct a 95% confidence interval for β_3 .
- (e) Perform the F -test for this model using a significance level of $\alpha = 0.05$.. State your null and alternative hypotheses, F -statistic, degrees of freedom of the numerator/denominator, p -value, and conclusion.