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MAT 222
Fall 2019
Homework 9

"Bread makes you fat?!"

–Scott Pilgrim, Scott Pilgrim vs. The World

Problem 1: To investigate the effects of musical genre on consumer spending, a study was conducted at a single high-end restaurant over a 3-week period. Each participant was subjected to a certain background music and their total food bill was recorded, all of which is summarized in the table below.

Background Music	n	Mean Bill, $\overline{x}$	Standard Deviation, s
Classical	21	29.921	2.781
Pop	24	27.171	3.257
None	18	26.904	4.132

A partially completed ANOVA table for this experiment is provided below:

Source	DF	SS	MS	F
Groups	*2*	*115.603*	*57.802*	*5.034*
Error	*60*	688.913	*11.482*	—
Total	*62*	804.516	—	—

(a) Complete the ANOVA table, rounding to 3 decimal places.

(b) Find the value of the pooled standard deviation, s_p. What conditions on the s do you need for s_p to be 'valid'? Is that condition met here?

We need the largest s to be smaller than twice the smallest s. Because $4.132 < 5.562$, the criterion is met here. Now $s_p = \sqrt{MSE} = \sqrt{11.482} = 3.3884$. Equivalently,

$$s_p^2 = \frac{(21-1)\,2.781^2 + (24-1)\,3.257^2 + (18-1)\,4.132^2}{(21-1)+(24-1)+(18-1)} = \frac{688.9126}{60} = 11.482$$

Therefore, $s_p = \sqrt{11.482} = 3.388$.

(c) State the null and alternative hypotheses to be examined with an F-test, and draw your conclusions at a 5% significance level.

$$\begin{cases} H_0 : \mu_{\text{classic}} = \mu_{\text{pop}} = \mu_{\text{none}} \\ H_a : \text{not all } \mu_i \text{ equal} \end{cases}$$

We have degrees of freedom $(2, 60)$. With F-statistic $F = 5.034$, we have $0.001 < p < 0.01$. Therefore, we reject the null hypothesis. Not all the means are the same.

(d) Suppose that the researcher would like to determine whether the population mean spending with no background music is significantly lower than the average of the mean dining bills with the other two background types. Using an appropriate contrast, carry out the test at $\alpha = 0.05$.

Because we are testing $\mu_{none} < \dfrac{\mu_{classic} + \mu_{pop}}{2}$*, which is the same as* $2\mu_{none} - \mu_{classic} - \mu_{pop} < 0$*, we have contrast* $c = 2\overline{x}_{none} - \overline{x}_{classic} - \overline{x}_{pop}$*. Then* $a_1 = 2$*,* $a_2 = -1$*,* $a_3 = -1$*. This gives*

$$SE_c = s_p\sqrt{\sum \frac{a_i^2}{n_i}} = 3.388\sqrt{\frac{2^2}{21} + \frac{(-1)^2}{24} + \frac{(-1)^2}{18}} = 1.817.$$

We test $H_0 : \psi = 0$ *against* $H_a : \psi < 0$*. We know* $c = 2(26.904) - 29.921 - 27.171 = -3.284$*. This gives test statistic* $t = c/SE_c = -3.284/1.817 = -1.807$*. With degrees of freedom* $(21-1) + (24-1) + (18-1) = 60$*, this gives* $0.025 < p < 0.05$*. Therefore, we reject the null hypothesis. The mean with no music is less than the average of the means with music.*

Problem 2: A 3×5 two-way ANOVA was run with 4 observations per cell (treatment group); that is, Factor A has 3 levels and Factor B has 5 levels.

(a) Specify the degrees of freedom of the numerator and denominator for the F-statistic which is used to test for the interaction in this analysis.

We have $n = 4 \cdot (3 \cdot 5) = 60$*. We have DFT=* $60 - 1 = 59$*, DFA=* $3 - 1 = 2$*, DFB=* $5 - 1 = 4$*, DFAB=* $2 \cdot 4 = 8$*, and DFE=* $60 - 3(5) = 45$*. Then we have degrees of freedom of the numerator* 8 *and degrees of freedom of the denominator* 45*.*

(b) The calculated value for the F-statistic for the interaction was 1.74. Find the corresponding p-value (or a range for the p-value), and state your conclusions at $\alpha = 0.05$.

We have $F = 1.74$ *with degrees of freedom* $(8, 45)$*. This gives us* $p > 0.10$*. Therefore, we fail to reject the null hypothesis. There is no interaction between factors* A *and* B*.*

(c) Would you expect the interaction plot of the cell means to look parallel? Explain.

Yes. When there is no interaction between the factors, the mean lines tend to be parallel.