Question 1 (Car Gas Mileage, Chapter 6) (12)

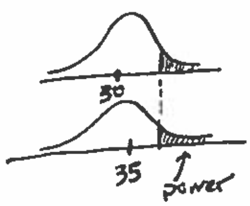
A car producer invents a new injection principle that helps to increasing the car gas mileage due to a more economical fuel consumption. The mean gas mileage with the old engine was $\mu = 30$ Miles/Gallon. The producer builds $n = 100$ cars with the new engine to test

$n = 100$
$\sigma = 10$ mpg
* $n \geq 30$ so CLT applies

$$H_0 \; : \; \mu = 30$$
$$H_a \; : \; \mu > 30$$

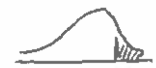
The test used rejects H_0 if $\bar{X} > 32$ and fails to reject (accepts) H_0 otherwise. You can assume that the car gas mileage data is normal distributed with a standard deviation of $\sigma = 10$ mpg.

	T	F
R	I_α	power
FR		II_β



(4)

1. What is the power of your test against the alternative $\mu = 35$?



$$z = \frac{32-35}{10/\sqrt{100}} = \frac{-3}{1} = -3.00 \rightsquigarrow 0.0013$$

$$1 - 0.0013 = 0.9987$$

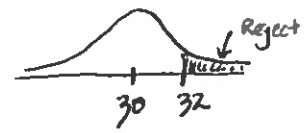
$\boxed{\text{power} = 0.9987}$

(4)

2. What is the probability of a type 1 error; that is, the probability that your test rejects H_0 when in fact $\mu = 30$?

Crit. Value:
$$z = \frac{32-30}{10/\sqrt{100}} = 2/1 = 2.00$$
Critical value: 2.00

$$P(\text{Type I error}) = \alpha = P(\text{reject } H_0 \mid H_0 \text{ True})$$



$$z = \frac{32-30}{10/\sqrt{100}} = \frac{2}{1} = 2.00 \rightsquigarrow 0.9772$$

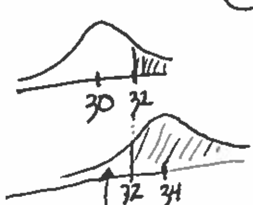
$$1 - 0.9772 = 0.0228$$

$\boxed{P(\text{Type I error}) = \text{Sig. level} = \alpha = 0.0228}$

(4)

3. What is the probability of a type 2 error, when $\mu = 34$?

* Can't use (a) because different alternative mean.



	T	F
R		
FR		II

$$Z = \frac{32-34}{10/\sqrt{100}} = \frac{-2}{1} = -2.00 \rightsquigarrow 0.0228$$

$\boxed{P(\text{Type II error}) = 0.0228}$

Question 2 (TV Consumption, Chapter 7) (10)

You read in a newspaper that the average television consumption in America is approximately $\mu = 80$ hours per month. You assume that students spend less time in front of the television. You ask $n = 6$ friends, all students, about their TV habits. They claim to watch

$$\{70, 65, 60, 75, 80, 70\}$$

hours of television per month.

(2)

1. Estimate the mean and the standard deviation.

   (a) $\bar{X} = 70$

   (b) $s = 7.07$

   $n = 6$

   Need sample to be approx normally dist. with no skewness or outliers.

   Perform a *One-Sample t-Test* to verify your assumption that students spend less time watching television than an average person.

(2)

2. State

$$\begin{cases} H_0 : \mu = 80 \\ H_a : \mu < 80 \end{cases}$$

dof 5

Critical value: -2.015

(2)

3. Calculate the t-statistic.

$$t = \frac{70 - 80}{7.07/\sqrt{6}} = \frac{-10}{2.886} = \boxed{-3.465}$$

test statistic

(2)

4. Approximate the P-value.

   dof = 6-1=5

$$t = -3.465 \xrightarrow{\text{dof } 5} \boxed{0.005 < p < 0.01}$$

   P-value range

Using critical value.
$t = -3.465 < -2.015$
so reject Ho.

(2)

5. Use an $\alpha = 0.05$ level to draw a conclusion.

   $p < \alpha$, reject Ho.

   There is sufficient evidence to suggest that Americans consume less than 80hrs/week watching television, on average.

3. (18 points) A British study compared 98 drivers and 93 conductors of London double-decker buses regarding daily calory consumptions. Some of the study results are summarized below:

$*\ n_D + n_C = 191 \geq 40$ so we are fine even if skewed.

	n	$\bar{x}$	s
Drivers	98	2821	435.6
Conductors	93	2844	437.3

$\alpha = 0.05$

(a) Is there a significant evidence at the 5% level that conductors consume more calories per day than do drivers? Assuming that the two population standard deviations are not equal, conduct the two-sample t-test. State your hypotheses, give a P-value, and draw your conclusion.

0.05, 5%

$$\begin{cases} H_0: \mu_C \neq \mu_D \\ H_a: \mu_C > \mu_D \end{cases}$$

$$t = \frac{2844 - 2821}{\sqrt{\frac{437.3^2}{93} + \frac{435.6^2}{98}}} = \frac{23}{63.186} = 0.364$$

$\}$ dof 80 $\rightarrow p > 0.25$

dof $= \min\{98-1, 93-1\} = 92$ (use 80)

$$\boxed{\begin{array}{l} \begin{cases} H_0: \mu_C = \mu_D \\ H_a: \mu_C > \mu_D \end{cases} \\ \text{Test statistic: } 0.364 \\ \text{Critical value: } 1.664 \\ \text{p-value: } p > 0.25 \\ \text{dof} = 92 \end{array}}$$

$p > \alpha$. Fail to reject H_0

The sample data is consistent with the null hypothesis that, on average, drivers and conductors consume the same amount of daily calories

Note: $0.364 \not> \underbrace{1.664}_{\text{crit. value}}$ so fail to reject

(b) Now, suppose that the use of the pooled two-sample t-test is justified for this data. If you use the pooled t-test, how would your test statistic change? (Write down the formula for your test statistic and calculate s_p, the pooled estimator of σ.) What is the degrees of freedom associated with your test statistic?

$*\ \frac{437.3}{435.6} = 1.00 < 2$ so pooling could be appropriate.

We need to change SE, using S_p.

The degrees of freedom change:

dof $= 98 + 93 - 2 = 189$

We have $S_p = 436.43$

The p-value should decrease (if only slightly).

dof $= 98 + 93 - 2 = 189$ (use 100)

$$S_p^2 = \frac{(98-1)435.6^2 + (93-1)437.3^2}{98+93-2} = 190469.69$$

$\rightarrow S_p = \sqrt{190469.69}$

$S_p = 436.43$

$$t = \frac{2844 - 2821}{436.43\sqrt{\frac{1}{93} + \frac{1}{98}}} = \frac{23}{63.18} = 0.364$$

$\}$ dof 100 $\rightarrow p > 0.25$

2. (14 pts) A blind experiment was conducted for 49 college students for their preference between Coke and Pepsi. Twenty nine students preferred Coke, while the rest chose Pepsi.

$n = 49$
$X = 29$
$\hat{p} = \frac{29}{49} = 0.5918$

(a) Find a 95% confidence interval for the proportion of college students who prefer Coke to Pepsi.

success

*At least 10 succ./failures

$$0.5918 \pm 1.960\sqrt{\frac{0.5918(1-0.5918)}{49}}$$
$$0.5918 \pm 1.960(0.0702)$$
$$0.5918 \pm 0.1376$$
$$(0.4542, 0.7294)$$

We are 95% certain than, on average, 45.42% to 72.94% of people prefer Coke to Pepsi.

(b) Do the results provide good evidence that a majority prefers one of the drinks? Explain why or why not.

For a majority to prefer one, 50% or more need to prefer one. But there is a 95% chance that 50% of people prefer Coke/Pepsi. (see conf. intv). Therefore, no one soda seems to be in the majority.

% or prop? Assume prop.

(c) How large a sample would be required to obtain a margin of error of ±0.10 in a 95% confidence interval for the proportion of those who prefers Coke?

No good evidence to suggest we know p. (So no p*). Then we have....

$$n \geq \frac{1}{4}\left(\frac{1.960}{0.10}\right)^2$$

$$n \geq 96.04 \rightsquigarrow 97$$

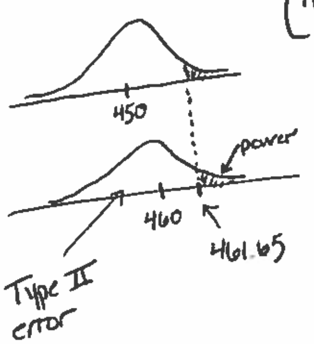
A sample of at least 97 people would need to be taken.

1. Consider the test H_0: $\mu = 450$ against the alternative hypothesis H_a: $\mu > 450$ at the α = 1% level of significance. Assume that the population standard deviation is $\sigma = 100$ and a simple random sample of size 400 is to be taken.
   a. (5 points) Calculate the critical value. For what values of the sample mean will H_0 be rejected?
   b. (5 points) Calculate the probability of type-II error against the alternative $\mu = 460$.
   c. (2 points) Calculate the power of the test against the alternative $\mu = 460$.

$\sigma = 100$

$n = 400$

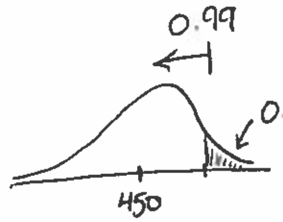
	T	F
R	I_α	
FR		β_{II}



* $n = 400 \geq 30$ so CLT applies

$$\begin{cases} H_0: \mu = 450 \\ H_a: \mu > 450 \end{cases}$$

a)



$z = 2.33$

Critical value: 2.33

$$2.33 = \frac{\bar{x} - 450}{100/\sqrt{400}}$$

$$2.33 = \frac{\bar{x} - 450}{5}$$

$$\bar{x} - 450 = 11.65$$

$$\bar{x} = 461.65$$

$\boxed{\text{Critical value: } 2.33 \quad \text{reject for... } \bar{x} \geq 461.65}$

P(Type I error)
= sig. level
= α
= 0.01

0.01	0.3707
0.99	0.6293

b) P(Type II) = P(Fail reject | H₀ F)

$$z = \frac{461.65 - 460}{100/\sqrt{400}} = \frac{1.65}{5} = 0.33 \rightsquigarrow 0.6293$$

$$\boxed{\beta = P(\text{Type II error}) = 0.6293}$$

c) *Note: uses same alt. mean

$$\text{Power} = 1 - P(\text{Type II error})$$
$$= 1 - 0.6293$$
$$= \boxed{0.3707}$$

2. A certain upper-level course for a particular major at a certain university is offered only to juniors and seniors. The college dean is interested in assessing whether the proportion of juniors that receive an A in that course is significantly less than the proportion of seniors that receive an A in that course. A random sample of students across different semesters was taken. The following table provides the summary of the data:

$X_J = 14 \quad n_j = 70$
$X_s = 24 \quad n_s = 80$
$\hat{p}_J = 0.20 ; \hat{p}_s = 0.30$

	Total No. of students	No. of students that received an A
Juniors	70	14
Seniors	80	24

*At least 5 suc/fail. per sample so good.

a. (9 points) Carry out an appropriate test at level of significance α=5% to answer the college dean's question. <u>The following details of the test must be provided</u>: the null & the alternative hypotheses, the test statistic, its p-value, the decision at α=5%, and a conclusion in context.

$$\begin{cases} H_0: p_s = p_j \\ H_a: p_s > p_j \end{cases}$$

Equiv, $p_J < p_s$

Crit value: 1.645
test stat: 1.40
p-value: 0.0808
$\alpha = 0.05$

$1.40 < 1.645$
fail to reject

$\hat{p} = \frac{14+24}{70+80} = 0.2533 ; \quad SE_{D_p} = \sqrt{0.2533(1-0.2533)\left(1/70 + 1/80\right)}$
$= 0.0712$

$z = \frac{0.30 - 0.20}{0.0712} = 1.40 \rightsquigarrow 0.9192$
$p = 0.0808$

$p > \alpha$. Fail to reject H_0

The data is consistent with the null hypothesis that the proportion of seniors and juniors receiving an A in the course are the same.

b. (8 points) Find a 90% confidence interval for the difference of the population proportions.

$\hat{p}_j = 0.20$
$\hat{p}_s = 0.30$
$z^* = 1.645$

$SE_D = \sqrt{\frac{0.20(1-0.20)}{70} + \frac{0.30(1-0.30)}{80}} = 0.0701$

$(0.30 - 0.20) \pm 1.645(0.0701)$

0.10 ± 0.12

$(-0.02, 0.22)$

We are 90% certain that, on average, the proportion of ~~students~~ seniors receiving an A in the course is between 0.02 less to 0.22 more than juniors receiving an A.

* Each sample had at least 5 suc./failures

3. In a Time/CNN survey, 49 of 205 single women said that they "definitely want to get married." In the same survey, 70 of 260 single men gave that same response.

   a. Find the sample proportion of single women that said they definitely want to get married and find its standard error. (5 points)

$X_F = 49$

$n_F = 205$

$\hat{p}_F = 0.2390$

$$\boxed{\begin{array}{l} \hat{p}_F = 0.2390 \\ SE_{\hat{p}_F} = \sqrt{\dfrac{0.2390(1-0.2390)}{205}} = 0.0298 \end{array}}$$

   b. Knowing that the sample proportion of "definitely want to get married" for single men is .27, and its standard error for the male data is .0275. Using the information, and the results from part (a), give a 90% confidence interval for the difference. (5 points)

$$(0.27 - 0.2390) \pm 1.645\sqrt{\underbrace{\frac{0.2390(1-0.2390)}{205}}_{0.0298^2} + \underbrace{\frac{0.27(1-0.27)}{260}}_{0.0275^2}}$$

$$0.031 \pm 1.645(0.0406)$$

$$0.031 \pm 0.067$$

$$(-0.036, 0.098)$$

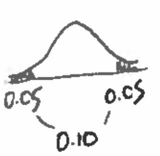
We are 90% confident that, on average, ~~men want~~ 3.6% less of men to 9.8% of men want to get married more than women.

   c. Use a test of significance to examine whether the two proportions are equal. State your null and alternative hypothesis, your test statistic, your p-value, and conclusion. (10 points)

$$\begin{cases} H_0: p_M = p_F \\ H_a: p_M \neq p_F \end{cases}$$

$$\hat{p} = \frac{49+70}{205+260} = 0.2559 \qquad \alpha = 0.10$$

$$SE_{D_p} = \sqrt{0.2559(1-0.2559)\left(\tfrac{1}{205} + \tfrac{1}{260}\right)} = 0.04076$$



Crit. values: ± 1.645

$$Z = \frac{0.27 - 0.2390}{0.04076} = \frac{0.031}{0.04076} = 0.76 \rightsquigarrow \frac{0.7764}{0.2236}$$

test statistic

$$P = 2(0.2236) = 0.4472 > \alpha$$

Fail to reject H_0.

The data is consistent with proportion of men and women wanting to get married being the same.

* $|z| \ngtr 1.645$ so know fail to reject

* $0 \in$ conf. intv. so fail to reject

Sign. level = 0.05

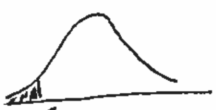
5. Does cocaine use by pregnant women cause their babies to have low birth weight? To study this question, birth weights of babies of women who tested positive for cocaine/crack during a drug-screening test were compared with the birth weights for women who either tested negative or were not tested, a group we call "other". Here are the summary statistics. The birth weights are measured in grams.

$*n_p + n_o = 6108 > 40$ so good.

Group	n	$\bar{x}$	s
Positive test	134	2733	599
Other	5974	3118	672

(a) Formulate appropriate hypotheses to answer the question.

(b) Assuming the two populations have different variances, use an appropriate test to test the null hypotheses that the mean cocaine use for the two groups are the same versus the alternative hypotheses that they are not the same.



0.05 dof 100

(c) Assuming the two populations have the same variance, use an appropriate test to test the null hypotheses that the mean cocaine use for the two groups are the same versus the alternative hypotheses that they are not the same.

Crit. value: −1.646

(d) Give a 95% confidence interval for the mean difference in birth weights.

a) $\begin{cases} H_0: \mu_p = \mu_o \\ H_a: \mu_p < \mu_o \end{cases}$

b) dof = min (134−1, 5974−1) = 133 ⟶ use 100

$$t = \frac{3118 - 2733}{\sqrt{\frac{672^2}{5974} + \frac{599^2}{134}}} = \frac{385}{52.471} = 7.337 \xrightarrow{\text{dof } 100} p < 0.0005$$

↑ test statistic

$*\ \frac{672}{599} = 1.12 < 2$ so pooling might be appropriate

c) dof = 134 + 5974 − 2 = 6106 ⟶ use 1000

$$S_p^2 = \frac{(134-1)599^2 + (5974-1)672^2}{134 + 5974 - 2} = 449563.01 \rightsquigarrow S_p = \sqrt{\quad} = 670.49$$

$$t = \frac{3118 - 2733}{670.49\sqrt{\frac{1}{134} + \frac{1}{5974}}} = \frac{385}{58.568} = 6.57 \xrightarrow{\text{dof } 1000} p < 0.0005$$

↑ test statistic

Conclusion: We reject H_0: There is significant evidence to suggest that children born from mothers having tested pos for cocaine/crack, on average, weigh less than 'other' babies

d) $(3118 - 2733) \pm 1.962(670.49) \rightsquigarrow 385 \pm 1315.5 \rightsquigarrow (-930.5, 1700.5)$

We are 95% certain that, on average, children born from 'neg' mothers weigh 930.5g less to 1700.5g more than those children born from 'pos' mothers.

1. Some books state the IQ of Americans aged 20-34 has a mean of 110, while other books state the mean IQ is 100. Suppose that, to settle this dispute, 150 Americans aged 20-34 are chosen at random, and their IQ's are measured. Take the null hypothesis to be

H_0: population mean IQ is 100, vs. the alternative hypothesis

H_a: population mean IQ is >100.

Assume the population standard deviation is known to equal 30. Reject the null hypothesis if the sample mean $\overline{X}$ found is greater than 105.

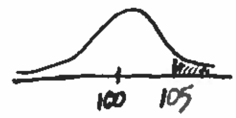
a. Find the level of significance of this test? (Find it as a number.) (6 points)

b. Find the Type II error probability of the test when the population mean is actually 110. (6 points)

$*\, n = 150 \geq 30$ so CLT applies

$\sigma = 30$

	T	F
R	I α	power
NR		β II

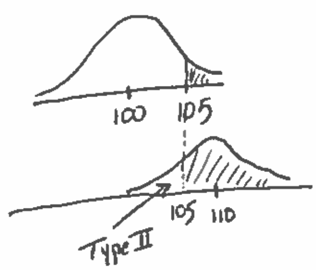


a) sig level $= \alpha =$ P(Type I error) $=$ P(reject H_0 | H_0 T)

$$Z = \frac{105-100}{30/\sqrt{150}} = \frac{5}{2.449} = 2.04 \rightsquigarrow \frac{\begin{array}{r}1\\ 0.9793\end{array}}{0.0207}$$

$$\boxed{\alpha = 0.0207}$$

b)



P(Type II error) $=$ P(Fail to reject | H_0 F)

$$Z = \frac{105-110}{30/\sqrt{150}} = \frac{-5}{2.449} = -2.04 \rightsquigarrow 0.0207$$

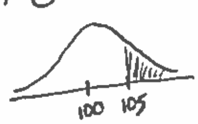
$$\boxed{P(\text{Type II error}) = 0.0207}$$

c) Power to detect $\mu = 110$

Power $= 1 -$ P(Type II error) $= 1 - 0.0207 = 0.9793$

d) Critical value

Crit. value $= 2.04$



$$Z = \frac{105-100}{30/\sqrt{150}} = \frac{5}{2.449} = 2.04$$

reject if $\dfrac{\bar{X}-100}{30/\sqrt{150}} \geq 2.04$, ie if $\bar{X} \geq 105$

1. A marketing campaign by an internet service provider (ISP) advertises internet download speeds of 15 Mbps (megabytes per second). A consumer research agency suspects that the speeds are less than what is claimed in the advertisements. A random sample of speeds from different times and different locations was taken. Summary statistics (sample size, sample mean, sample standard deviation) of the variable "DLspeed" (internet download speed) from the sample is given below:

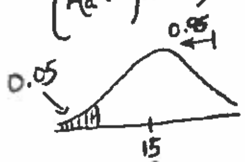
Descriptive Statistics: DLspeed

Variable	Total Count	Mean	StDev
DLspeed	30	14.15	2.16

* $n=30$
$15<n<40$
So data needs no skewness or outliers

a. (9 points) Using an appropriate test at level of significance α=5%, assess whether the population mean download speed is smaller than advertised by the ISP. <u>The following details of the test must be provided</u>: the null & the alternative hypotheses, the test statistic, its degrees of freedom, its p-value range, decision at α=5%, and a conclusion in context.

$\begin{cases} H_0: \mu = 15\text{Mbps} \\ H_a: \mu < 15\text{Mbps} \end{cases}$



dof 29

$\boxed{\text{Crit value: } -1.699 \quad t=-2.157<-1.699 \text{ so reject } H_0}$

dof $= 30-1 = 29$

$t = \dfrac{14.15-15}{2.16/\sqrt{30}} = \dfrac{-0.85}{0.394} = -2.157 \xrightarrow{\text{dof }29} 0.01<p<0.02$

test statistic

$p<\alpha$. Reject H_0

There is sufficient evidence to suggest that the average download speed for this ISP is less than 15Mbps.

$\boxed{\text{test stat: } -2.157 \quad \text{dof: } 29 \quad \text{p-range } 0.01<p<0.02}$

b. (5 points) Find a 98% confidence interval for the population mean download speed. Clearly indicate the t* used.

dof $=29$

$t^* = t_{\alpha/2} = 2.462$

$14.15 \pm 2.462 \dfrac{2.16}{\sqrt{30}}$

14.15 ± 0.97

$(13.18, 15.12)$

We are 98% confident that the average download speed for this ISP is between 13.18Mbps and 15.12Mbps.

2. A study was conducted to understand the difference in blood pressure between people with two different types (High vs. Low) of sleeping habit. The data from the study is below:

$\frac{7.16}{6.77} = 1.05 < 2$
So pooling could be appropriate.

Sleep Group	Count	Sample Mean	Sample Standard Deviation
High	5	111.40	6.77
Low	5	120.20	7.16

$* n_1 + n_2 = 10$
$10 < 15$
Need both samples approx normal

a. (7 points) Is the mean blood pressure higher for people in the low sleep group? Carry-out an appropriate test. State the null & alternate hypotheses, calculate the corresponding test statistic, state its degrees of freedom, and its p-value.
b. (3 points) Is there significant evidence that the mean blood pressure is higher for people in the low sleep group at level 10%? At level 5%? At level 1%?
c. (5 points) Give a 99% confidence interval for the mean difference in mean blood pressure between the sleep groups.

Pooled

a) $\begin{cases} H_0: \mu_L = \mu_H \\ H_0: \mu_L > \mu_H \end{cases}$

dof $= 5+5-2 = 8$

$\alpha = 0.05$

Crit value: 1.860

$S_p^2 = \frac{(5-1)6.77^2 + (5-1)7.16^2}{5+5-2} = 48.549$

$1.997 > 1.860$ reject H_0

$S_p = \sqrt{-} = 6.967$

$t = \frac{120.20 - 111.40}{6.967\sqrt{1/5 + 1/5}} = \frac{8.8}{4.406} = 1.997 \xrightarrow{dof 8} 0.025 < p < 0.05$

test stat p-range

Reject H_0. There is sufficient evidence to suggest that the avg blood pressure in low sleep group is higher than that of those in high sleep group.

b)

α	0.10	0.05	0.01
Dec.	reject	reject	Fail to reject

c) $(120.20 - 111.40) \pm 3.355\,(4.406)$

8.8 ± 14.78

$(-5.98, 23.58)$

Unpooled

a) $\begin{cases} H_0: \mu_L = \mu_H \\ H_a: \mu_L > \mu_H \end{cases}$

dof $= \min(5-1, 5-1) = 4$

$\alpha = 0.05$

Crit value: 2.132

$\sqrt{\frac{6.77^2}{5} + \frac{7.16^2}{5}} = \sqrt{19.4197} = 4.407$

$t = \frac{120.20 - 111.40}{4.407} = \frac{8.8}{4.407} = 1.996$

dof 4

$0.05 < p < 0.10$

$1.996 < 2.132$
Fail to reject

Fail to reject H_0. The data is consistent with the null hypothesis that the blood pressure is the same between the two types of sleep types examined.

b)

α	0.10	0.05	0.01
Dec	Reject	Fail	Fail

c) $(120.20 - 111.40) \pm 4.604\,(4.407)$

8.8 ± 20.29

$(-11.49, 29.09)$

2. (14 points) A software developer is interested in analyzing the proportion of CPAs who use a certain accounting software.

(a) How many observations should be taken to estimate, at 95% confidence level, the population proportion within the margin of error of 0.03? Use the conservative approach assuming that no prior information about the proportion is available.

95% conf.

$Z^* = Z_{\alpha/2} = 1.960$

$m = 0.03$

$n = \frac{1}{4}\left(\frac{1.960}{0.03}\right)^2$

$n = 1067.11 \rightarrow 1068$

At least 1,068 observations should be taken.

If had reason to believe $p = 0.38$

$p^* = 0.38$ $m = 0.03$

$Z^* = 1.960$

$n = \left(\frac{1.960}{0.03}\right)^2 0.38(1-0.38) = 1005.64 \rightarrow 1006$

At least 1,006 observations should be used.

(b) A random sample of 200 CPAs was collected and eighty nine CPAs out of 200 in the sample reported using the specific software. Find a 95% confidence interval for the true proportion of CPAs who use that accounting software.

* At least 10 suc/failure so good.

$X = 89$

$n = 200$

$\hat{p} = 0.445$

$Z^* = 1.960$

$0.445 \pm 1.960\sqrt{\frac{0.445(1-0.445)}{200}}$

0.445 ± 0.069

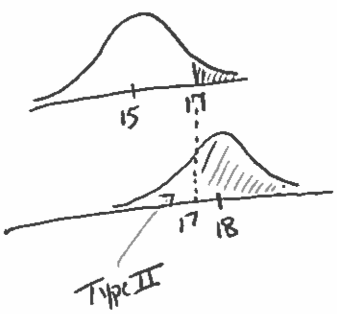
$(0.376, 0.514)$

We are 95% certain that between 37.6% and 51.4% of CPAs use the software.

*$n=10$, so need org. dist. normally dist.

$\sigma=4$

	T	F
R	α I	power
FR		β II



1. (15 points) The attention span of little kids (ages 3-5) is claimed to be Normally distributed with a mean of 15 minutes and a standard deviation of 4 minutes. A test is to be performed to decide if the average attention span of these kids is really this short, or if it is longer. To test the hypotheses $H_0 : \mu = 15$ versus $H_a : \mu > 15$, you plan to select a SRS of 10 children who will watch a TV show they have never seen before and to record the time until they walk away from the show. Suppose that you decide to reject the null hypothesis if the observed sample mean is greater than 17 minutes.

   (a) Find the probability of a Type I error for this test.

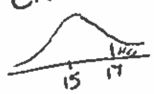
   $\begin{cases} H_0: \mu=15 \\ H_a: \mu>15 \end{cases}$

   $\alpha = P(\text{Type I error}) = P(\text{reject } H_0 \mid H_0\ T)$

   $Z = \dfrac{17-15}{4/\sqrt{10}} = \dfrac{2}{1.265} = 1.58 \leadsto \dfrac{1 - 0.9429}{0.0571}$

   $\boxed{P(\text{Type I error}) = 0.0571}$

   Crit. value:

   $Z = \dfrac{17-15}{4/\sqrt{10}} = \dfrac{2}{1.265} = 1.58$

   reject if $\bar{x} \geq 17$ or if $\dfrac{\bar{x}-15}{4/\sqrt{10}} \geq 1.58$

   (b) Find the probability of a Type II error if the true mean is 18 minutes.

   $P(\text{Type II error}) = P(\text{fail to reject} \mid H_0\ F)$

   $Z = \dfrac{17-18}{4/\sqrt{10}} = \dfrac{-1}{1.265} = -0.79 \leadsto 0.2148$

   $\beta = \boxed{P(\text{Type II error}) = 0.2148}$

   b2) Power to detect $\mu=18$

   power $= 1 - P(\text{Type II error}) = 1 - 0.2148 = \boxed{0.7852}$

   (c) Based on your answer in (b), do you think this test has adequate power? Why or why not? If not, how can you increase the power?

   Typically, we want power ≥ 0.80. But this is close. There is 78.52% we reject H_0 if $\mu=18$. If we wanted to increase power...

   - Increase α
   - Consider μ_{alt} farther from μ_0
   - Increase n
   - Decrease σ

2. The Medassist Pharmaceutical Company wants to test Dozenol, a new cold medicine intended for night use. Tests for such products often include a "treatment group" of people who use the drug and a "control group" of people who don't use the drug. Fifteen people with colds are given Dozenol and 30 others are not given. The systolic blood pressure is measured for each subject, and the sample statistics are given below.

$* n_1 + n_2 = 45 > 40$ so good

	n	$\overline{x}$	s
Treatment Group	15	203.4	19.4
Control Group	30	189.4	19

(a) Find a 95% confidence interval for the change of the blood pressure after taking Dozenol.
(b) Use the significance level 0:05 to test the claim that the population means for treatment group and the control group are equal.
(5 points each)

$* s_T \approx s_C$, ie $\frac{19.4}{19} = 1.02 < 2$ so pooling definitely appropriate

a) dof $= 15 + 30 - 2 = 43$ (use 40)

$$s_p^2 = \frac{(15-1)19.4^2 + (30-1)19^2}{15+30-2} = 366.0009 \rightsquigarrow s_p = \sqrt{\sim} = 19.13$$

$t^* = 2.021$

$$(203.4 - 189.4) \pm (2.021)\, 19.13\sqrt{\tfrac{1}{15} + \tfrac{1}{30}}$$

$$14 \pm 12.23$$

$$(1.77, 26.23)$$

We are 95% certain that, on average, the blood pressure of patients on Dozenol is 1.77 to 26.23 higher than those not on the medication.

b) $\alpha = 0.05$

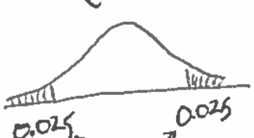
$\begin{cases} H_0: \mu_T = \mu_C \\ H_a: \mu_T \neq \mu_C \end{cases}$

$$t = \frac{203.4 - 189.4}{19.13\sqrt{\frac{1}{15} + \frac{1}{30}}} = \frac{14}{6.049} = 2.314 \leftarrow \text{test statistic}$$

dof 40

$0.01 < p < 0.02$

$\times 2$ $0.02 < p < 0.04$



dof 40

Critical value: ± 2.021

$p < \alpha$ Reject H_0

There is sufficient evidence to suggest that the blood pressure of those on Dozenol is higher than those not on the drug.

0 not in 95% C.I. (1.77, 26.23). Therefore, we reject H_0.

$t = 2.314 > \underset{\text{crit.}}{2.021}$ Therefore, we reject H_0.

1. Citing the results of a survey by the New York Times, a TV newscast claimed that 46% of New Yorkers were in favor of a proposed transportation bond (to be voted on in the upcoming election). When the pollsters at Marist College surveyed 360 New Yorkers, they found 184 who favored the transportation bond.

* At least 10 suc/failures so good

(a) What proportion of citizens favoring the transportation bond was observed in the Marist survey?

$X = 184$
$n = 360$

$$\hat{p} = \frac{184}{360} = 0.5111$$

(b) State hypotheses for an appropriate <u>one-tailed</u> test of the newscast's claim based on the Marist observations.

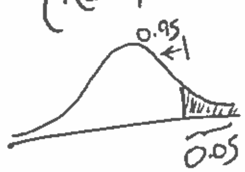
$$\begin{cases} H_0: p = 0.46 \\ H_a: p > 0.46 \end{cases}$$

$$\begin{cases} H_0: p = 0.46 \\ H_a: p > 0.46 \end{cases}$$

(c) Test the hypotheses from part (b). Give the *p*-value of your test, and using the .05 level of significance, state your conclusion regarding the claim made in the newscast.

$\alpha = 0.05$

$$\begin{cases} H_0: p = 0.46 \\ H_a: p > 0.46 \end{cases}$$



$\hat{p} = 0.5111$

test statistic

$$Z = \frac{0.5111 - 0.46}{\sqrt{\frac{0.46(1-0.46)}{360}}} = \frac{0.0511}{0.0263} = 1.94$$

0.9738

Crit. value: 1.645

* $Z = 1.94 > 1.645$ so reject H_0

$p = 0.0262$

$$\begin{array}{r} 1.000 \\ -\ 0.9738 \\ \hline p = 0.0262 \end{array}$$

$p < \alpha$
Reject H_0

There is sufficient evidence to suggest that proportion of New Yorkers in favor of proposed trans. bond is greater than 0.46.

Sign. level 0.01

*At least 5 suc./fail per sample so good.

4. A test question is considered good if it differentiates between prepared and unprepared students. The first question on a test was answered correctly by 62 of 80 prepared students and by 26 of 50 unprepared students. Perform a significance test for the null hypothesis $H_0 : p_1 = p_2$ versus $H_a : p_1 > p_2$ where p_1 is the population proportion of prepared students and p_2 is the population proportion of unprepared students. What do you conclude?

Prepared

$X_p = 62$

$n_p = 80$

$\hat{p}_p = \frac{62}{80} = 0.775$

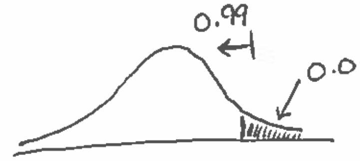
Unprepared

$X_u = 26$

$n_u = 50$

$\hat{p}_u = 26/50 = 0.52$

$$\begin{cases} H_0: p_p = p_u \\ H_a: p_p > p_u \end{cases}$$



Crit. value: 2.33

$$\hat{p} = \frac{62+26}{80+50} = 0.6769$$

$$SE_{D_p} = \sqrt{0.6769(1-0.6769)\left(\frac{1}{80}+\frac{1}{50}\right)}$$

$$= 0.08431$$

$$Z = \frac{0.775-0.52}{0.08431} = \underset{\text{test statistic}}{3.02} \rightsquigarrow \frac{1}{\geq 0.9987} \quad \underset{\text{p-value}}{0.0013}$$

$\alpha = 0.01$

test stat: 3.02

p-value: 0.0013

crit value: 2.33

Reject H_0

$P = 0.0013$

$P < \alpha$. Reject H_0

There is sufficient evidence to suggest that the proportion of prepared students answering the 1st question correctly is greater than the prop. of students unprepared answering it correctly.

$Z = 3.02 > \underbrace{2.33}_{\text{crit. value}}$ so we reject H_0.

2. (14 pts) A survey reported that 32 of 65 private schools in Ohio used standardized tests. The same survey reported that 40 of 100 private schools in New York used standardized tests. Treat these two groups as SRSs of private schools in Ohio and New York.

*At least 5 suc/failures per sample

(a) Find the two sample proportions.

$\hat{p}_O = \frac{32}{65} = 0.4923$ $\hat{p}_N = \frac{40}{100} = 0.40$

(b) Give a 99% confidence interval for the difference between the two proportions.

$z^* = 2.576$ $(0.4923 - 0.40) \pm 2.576\sqrt{\frac{0.4923(1-0.4923)}{65} + \frac{0.40(1-0.40)}{100}}$

$0.0923 \pm 2.576(0.0790)$

0.0923 ± 0.2035

$(-0.1112, 0.2958)$

We are 99% sure that the percentage of private schools in Ohio use stand. tests between 11.12% less to 29.58% more than private schools in NY.

(c) If you were asked to conduct a significance test for the difference between the two proportions, how would you determine the standard error required? Do not do the computations, but show all formulas and substitutions needed.

$X_1 = 32$

$X_2 = 40$

$n_1 = 65$

$n_2 = 100$

$SE_{D_p} = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$

$\hat{p} = \frac{X_1 + X_2}{n_1 + n_2}$