

**Math 222: Exam 1**  
**Spring – 2019**  
**02/22/2019**  
**50 Minutes**

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**Name:** \_\_\_\_\_

Write your name on the appropriate line on the exam cover sheet. This exam contains 8 pages (including this cover page) and 4 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

Question	Points	Score
1	25	
2	25	
3	25	
4	25	
Total:	100	

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1. (25 points) Mark each of the following statements as True (T) or False (F).
- (a) \_\_\_\_\_ A Type I error is rejecting a true null hypothesis.
  - (b) \_\_\_\_\_ A Type II error is rejecting a false null hypothesis.
  - (c) \_\_\_\_\_ Decreasing the confidence level for a test increases the corresponding probability of a Type II error.
  - (d) \_\_\_\_\_ Increasing the significance level decreases the probability of a Type II error.
  - (e) \_\_\_\_\_ Lowering the confidence level increases the margin of error.
  - (f) \_\_\_\_\_ Failing to reject  $H_0 : p = 0.37$  in a hypothesis test means that  $p = 0.37$ .
  - (g) \_\_\_\_\_  $t$ -tests are robust against violations of non-normality.
  - (h) \_\_\_\_\_ For  $t$ -procedures, it is more important that the sample be a SRS than it is for the data to be (approximately) normal.
  - (i) \_\_\_\_\_ If  $\mu_0 = 17.3$  and one finds  $\bar{x} = 19.4$ , then one must test  $H_0 : \mu_0 = 17.3$  against  $H_a : \bar{x} \geq 17.3$ .
  - (j) \_\_\_\_\_ You can decide whether to reject the null hypothesis for a one-sided test with significance level by using a confidence interval with confidence level  $C = 1 - \alpha$ .

2. Staff at the trauma ward of a hospital are trying to determine if recent policy changes have helped patients having undergone major surgery/trauma recover and leave the hospital quicker. They choose a SRS of 56 patients and measure their average post-surgery hospital stay,  $\bar{x}$ . To determine if the average hospitalization time has decreased from the average stay of 16.3 days two years ago, they test

$$H_0 : \mu = 16.3 \quad \text{against} \quad H_a : \mu < 16.3.$$

The staff decides on a test that rejects  $H_0$  if  $\bar{x} \leq 15.34$  days. Assume the standard deviation is  $\sigma = 4.1$  days.

- (a) (6 points) Find the probability of a Type I error for this test. What is the significance level for this test?

- (b) (7 points) Find the probability of a Type II error for this test if  $\mu = 14.5$ .

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(c) (6 points) What is the power for this test to detect  $\mu = 14.5$ ?

(d) (6 points) List three ways one could increase the power of this test.

3. NARCAN is a life saving nasal spray that can help reverse an opioid overdose. A journalist is interested in estimating the percentage of police officers that carry the life saving drug for when they encounter individuals who have overdosed. The journalist will collect data on Philadelphia police officers to make an estimate of police officers in larger cities who carry NARCAN.

(a) (6 points) If the proportion of officers in large cities who carry NARCAN is thought to be approximately 20%, how many police officers must the journalist examine to determine the true proportion within 3% and 90% certainty?

(b) (6 points) Regardless of the actual proportion of officers who carry NARCAN, how many officers must be examined to determine the true proportion who carry NARCAN within 3% and 90% certainty?

- (c) (7 points) The journalist finds that 1,005 of the approximately 6,700 officers in Philadelphia carry NARCAN. Construct a 90% confidence interval for the proportion of officers who carry NARCAN.

- (d) (6 points) Use the confidence interval from (c) to test the following hypothesis with  $\alpha = 0.10$ .

$$\begin{cases} H_0 : p = 0.20 \\ H_a : p \neq 0.20 \end{cases}$$

4. The African Bombardier beetle contains hydroquinones in its abdomen that it can spray causing a hot volatile 'explosive' burning stream. A researcher examines 49 male and 56 females beetles, and collects data on the amount of volatile liquid they contain. The researcher finds from the sample that, on average, male beetles contain 220 mg of liquid (with standard deviation 21.1 mg) while female beetles contain 211 mg of liquid (with standard deviation 8.8 mg).

(a) (6 points) Would a pooled  $t$ -test for the difference be appropriate here? Explain why or why not. Be sure to justify your answer mathematically.

(b) (6 points) Find a 99% confidence interval of difference of the average amount of volatile liquid male and female beetles contain.

- (c) (6 points) Test the following hypothesis at a significance level of  $\alpha = 0.01$ . Be sure to give your test statistic,  $p$ -value, and conclusion.

$$\begin{cases} H_0 : \mu_M = \mu_F \\ H_a : \mu_M > \mu_F \end{cases}$$

- (d) (7 points) Calculate the  $t$ -statistic if the hypothesized difference in the means was instead 2 mg.