**Math 222: Exam 1**
**Spring – 2019**
**02/22/2019**
**50 Minutes**

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Write your name on the appropriate line on the exam cover sheet. This exam contains 8 pages (including this cover page) and 4 questions. Check that you have every page of the exam. Answer the questions in the spaces provided on the question sheets. Be sure to answer every part of each question and show all your work. If you run out of room for an answer, continue on the back of the page — being sure to indicate the problem number.

| Question | Points | Score |
|---|---|---|
| 1 | 25 | |
| 2 | 25 | |
| 3 | 25 | |
| 4 | 25 | |
| Total: | 100 | |

1. (25 points) Mark each of the following statements as True (T) or False (F).

   (a) ____T____ A Type I error is rejecting a true null hypothesis.

   (b) ____F____ A Type II error is rejecting a false null hypothesis.

   (c) ____F____ Decreasing the confidence level for a test increases the corresponding probability of a Type II error.

   (d) ____T____ Increasing the significance level decreases the probability of a Type II error.

   (e) ____F____ Lowering the confidence level increases the margin of error.

   (f) ____F____ Failing to reject $H_0 : p = 0.37$ in a hypothesis test means that $p = 0.37$.

   (g) ____T____ $t$-tests are robust against violations of non-normality.

   (h) ____T____ For $t$-procedures, it is more important that the sample be a SRS than it is for the data to be (approximately) normal.

   (i) ____F____ If $\mu_0 = 17.3$ and one finds $\overline{x} = 19.4$, then one must test $H_0 : \mu_0 = 17.3$ against $H_a : \overline{x} \geq 17.3$.

   (j) ____F____ You can decide whether to reject the null hypothesis for a one-sided test with significance level by using a confidence interval with confidence level $C = 1 - \alpha$.

2. Staff at the trauma ward of a hospital are trying to determine if recent policy changes have helped patients having undergone major surgery/trauma recover and leave the hospital quicker. They choose a SRS of 56 patients and measure their average post-surgery hospital stay, $\overline{x}$. To determine if the average hospitalization time has decreased from the average stay of 16.3 days two years ago, they test

$$H_0 : \mu = 16.3 \quad \text{against} \quad H_a : \mu < 16.3.$$

The staff decides on a test that rejects $H_0$ if $\overline{x} \leq 15.34$ days. Assume the standard deviation is $\sigma = 4.1$ days.

(a) (6 points) Find the probability of a Type I error for this test. What is the significance level for this test?

*Assuming $H_0$ is true, we reject if $\overline{x} \leq 15.34$. But the probability of this is...*

$$z = \frac{15.34 - 16.3}{4.1/\sqrt{56}} = \frac{-0.96}{0.5479} = -1.75 \rightsquigarrow 0.0401$$

*Therefore, $P(\text{Type I Error}) = 0.0401$. Finally, we know $P(\text{Type I Error}) = \alpha$ so that the significance level is $\alpha = 0.0401$.*

(b) (7 points) Find the probability of a Type II error for this test if $\mu = 14.5$.

*We fail to reject if $\overline{x} \geq 15.34$. But if $\mu = 14.5$, then $P(\overline{x} \leq 15.34)$ is*

$$z = \frac{15.34 - 14.5}{4.1/\sqrt{56}} = \frac{0.84}{0.5479} = 1.53 \rightsquigarrow 0.9370.$$

*Therefore,*

$$P(\text{Type II Error}) = P(\overline{x} \geq 15.34) = 1 - P(\overline{x} \leq 15.34) = 0.0630.$$

(c) (6 points) What is the power for this test to detect $\mu = 14.5$?

*We computed this in the previous part: Power=* $P(\overline{x} \leq 15.34) = 0.9370$. *Alternatively, Power* $= 1 - P($*Type II Error*$) = 1 - 0.0630 = 0.9370$.

(d) (6 points) List three ways one could increase the power of this test.

- *Increase the significance level, i.e. choose* $\alpha$ *larger.*
- *Choose* $\mu_{alt}$ *farther from* $\mu_0$.
- *Increase the sample size.*
- *Decrease the standard deviation,* $\sigma$.

3. NARCAN is a life saving nasal spray that can help reverse an opioid overdose. A journalist is interested in estimating the percentage of police officers that carry the life saving drug for when they encounter individuals who have overdosed. The journalist will collect data on Philadelphia police officers to make an estimate of police officers in larger cities who carry NARCAN.

   (a) (6 points) If the proportion of officers in large cities who carry NARCAN is thought to be approximately 20%, how many police officers must the journalist examine to determine the true proportion within 3% and 90% certainty?

   *We are using a confidence level of 90% so that* $z^* = 1.645$. *We have also* $m = 0.03$ *and* $p^* = 0.20$. *Then*

$$n \approx \left(\frac{1.645}{0.03}\right)^2 0.20(1 - 0.20) = 481.071.$$

   *Therefore, at least* $n = 482$ *officers must be sampled.*

   (b) (6 points) Regardless of the actual proportion of officers who carry NARCAN, how many officers must be examined to determine the true proportion who carry NARCAN within 3% and 90% certainty?

   *The greatest error occurs when* $p^* = 0.50$. *Again, we have* $z^* = 1.645$, $m = 0.03$. *Then*

$$n \approx \frac{1}{4}\left(\frac{1.645}{0.03}\right)^2 = 751.674.$$

   *Therefore, at least* $n = 752$ *officers must be sampled.*

(c) (7 points) The journalist finds that 1,005 of the approximately 6,700 officers in Philadelphia carry NARCAN. Construct a 90% confidence interval for the proportion of officers who carry NARCAN.

*We have* $n = 6700$ *and* $\hat{p} = X/n = 1005/6700 = 0.15$. *For a 90% confidence interval,* $z^* = 1.645$. *Then we compute*

$$\begin{aligned} &\hat{p} \pm z^* SE_{\hat{p}} \\ &\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \\ 0.15 \pm 1.645&\sqrt{\frac{0.15(1-0.15)}{6700}} \\ 0.15 \pm 1.645&(0.00436) \\ &0.15 \pm 0.007 \end{aligned}$$

*The confidence interval is then* $(0.143, 0.157)$. *Therefore, we are 90% certain that between 14.3% and 15.7% of officers in large cities carry NARCAN.*

(d) (6 points) Use the confidence interval from (c) to test the following hypothesis with $\alpha = 0.10$.

$$\begin{cases} H_0 : p = 0.20 \\ H_a : p \neq 0.20 \end{cases}$$

*From (c), we have a 90% confidence interval (which corresponds to* $\alpha = 0.10$*) of* $(0.143, 0.157)$. *Now as* $p$ *is not in the interval* $(0.143, 0.157)$*, we know the* $p$*-value is at most* $0.10$*, so that we reject the null hypothesis. Therefore, it is not likely that approximately 20% of officers in large cities carry NARCAN.*

4. The African Bombardier beetle contains hydroquionens in its abdomen that it can spray causing a hot volatile 'explosive' burning stream. A researcher examines 49 male and 56 females beetles, and collects data on the amount of volatile liquid they contain. The researcher finds from the sample that, on average, male beetles contain 220 mg of liquid (with standard deviation 21.1 mg) while female beetles contain 211 mg of liquid (with standard deviation 8.8 mg).

   (a) (6 points) Would a pooled $t$-test for the difference be appropriate here? Explain why or why not. Be sure to justify your answer mathematically.

   *No, the pooled $t$-test would not be appropriate. The standard deviations are not sufficiently 'similar' as $s_M/s_F = 21.1/8.8 = 2.398$ is not between $0.50$ and $2$.*

   (b) (6 points) Find a 99% confidence interval of difference of the average amount of volatile liquid male and female beetles contain.

   *We summarize the data as follows:*

$$\begin{aligned} n_M &= 49 & n_F &= 56 \\ \overline{x}_M &= 220 & \overline{x}_F &= 211 \\ s_M &= 21.1 & s_F &= 8.8 \end{aligned}$$

   *We have degrees of freedom $\min(49-1, 56-1) = 48$, so that we are forced to use dof $= 40$ on Table D. Then we have $t^* = 2.704$. Therefore, we compute*

$$\begin{aligned} (\overline{x}_M - \overline{x}_F) &\pm t^* \sqrt{\frac{s_M^2}{n_M} + \frac{s_F^2}{n_F}} \\ (220 - 211) &\pm 2.704 \sqrt{\frac{21.1^2}{49} + \frac{8.8^2}{56}} \\ 9 &\pm 2.704(3.236) \\ 9 &\pm 8.75. \end{aligned}$$

   *Then the confidence interval is $(0.25, 17.75)$. Therefore, we are 99% certain that, on average, male beetles contain 0.25 mg to 17.75 mg more volatile liquid than female beetles.*

(c) (6 points) Test the following hypothesis at a significance level of $\alpha = 0.01$. Be sure to give your test statistic, $p$-value, and conclusion.

$$\begin{cases} H_0 : \mu_M = \mu_F \\ H_a : \mu_M > \mu_F \end{cases}$$

*We have $t$-statistic*

$$t = \frac{\overline{x}_M - \overline{x}_F}{\sqrt{\frac{s_M^2}{n_M} + \frac{s_F^2}{n_F}}} = \frac{9}{3.236} = 2.781 \overset{\text{dof } 40}{\rightsquigarrow} 0.005.$$

*Therefore, we reject the null hypothesis. It is more likely that male beetles, on average, contain more volatile liquid than female beetles.*

(d) (7 points) Calculate the $t$-statistic if the hypothesized difference in the means was instead 2 mg.

*Before, we had numerator $(\overline{x}_M - \overline{x}_F) - 0$ because the null hypothesized difference was $\mu_M - \mu_F = 0$. But now we suppose the difference is 2 mg. Therefore, we have*

$$t = \frac{(\overline{x}_M - \overline{x}_F) - \mu_{\textit{Diff}}}{\sqrt{\frac{s_M^2}{n_M} + \frac{s_F^2}{n_F}}} = \frac{9-2}{3.236} = 2.163.$$

*Notice this indicates a $p$-value of $0.02$, so that in this case we would fail to reject the null hypothesis, i.e. the data indicates that there is an approximately 2 mg difference in the volatile liquids for male and female beetles.*