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**MAT 222**
**Spring 2019**
**Chapter 10, 11 Worksheet**

*"It's not easy being drunk all the time. If it were, everyone would do it."*
*– Tyrion Lannister, Game of Thrones*

**Problem 1:** Concrete is a commonly used material in Civil Engineering. Comprehensive strength measures the ability of concrete materials to endure various strains. Researchers examined 1,030 concrete samples and attempted to try to predict the comprehensive strength of various mixtures of concrete using the cement amount, blast furnace slag, fly ash, water, superplasticizer, coarse aggregate, and fine aggregate used in the construction as well as the age of the concrete.[1] Fill in the missing entries from the analysis of their model in the table below.

Analysis of Variance

| Source | DF | Adj SS | Adj MS | F-Value | P-Value |
|---|---|---|---|---|---|
| Regression | *8* | 176745 | *22093.1* | 204.27 | 0.000 |
| Cement | *1* | 21533 | 21533.3 | 199.09 | 0.000 |
| BF | *1* | 11353 | 11352.5 | *104.96* | 0.000 |
| FlyAsh | *1* | 5281 | 5281.3 | 48.83 | 0.000 |
| Water | *1* | *1513* | 1513.4 | 13.99 | 0.000 |
| SP | *1* | 1046 | 1046.3 | 9.67 | 0.000 |
| CoarseAg | *1* | 398 | *398.4* | 3.68 | 0.000 |
| FineAg | *1* | 384 | 383.5 | 3.55 | 0.000 |
| Age | *1* | 47905 | 47905.2 | 442.92 | 0.000 |
| Error | *1021* | *110428* | *108.2* | | |
| Total | *1029* | *287173* | | | |

Model Summary

| S | R-sq | R-sq (adj) | R-sq (pred) |
|---|---|---|---|
| *10.402* | *61.54%* | *61.54%* | 60.68% |

Coefficients

| Term | Coef | SE Coef | T-Value | P-Value | VIF |
|---|---|---|---|---|---|
| Constant | *-23.2* | 26.6 | -0.87 | 0.384 | |
| Cement | 0.11979 | 0.00849 | 14.11 | 0.000 | 7.49 |
| BF | 0.1038 | *0.0101* | 10.28 | 0.000 | 7.28 |
| FlyAsh | 0.0879 | 0.0126 | 6.99 | 0.000 | 6.17 |
| Water | -0.1503 | 0.0402 | -3.74 | *0.000* | 7.00 |
| SP | 0.2907 | 0.0935 | 3.11 | 0.002 | 2.97 |
| CoarseAg | 0.01803 | 0.00939 | *1.92* | 0.055 | 5.08 |
| FineAg | 0.0201 | 0.0107 | 1.88 | 0.060 | 7.01 |
| Age | *0.11423* | 0.00543 | 21.05 | 0.000 | 1.12 |

The regression equation is

Compression Strength $= -23.2 + 0.11979C + 0.1038B + 0.0879F - 0.1503W + 0.2907S + 0.01803CA + 0.0201FAg + 0.11423A$

[1] I–Cheng, Yeh, "Modeling of strength of high performance concrete using artificial neural networks.", *Cement and Concrete Research*, Vol. 28, No. 12, pp.1797–1808 (1998).

Furthermore, based on the data from the table, answer the following questions:

(a) What is the correlation coefficient?

$$R = \sqrt{R^2} = \sqrt{0.6154} = 0.7845$$

(b) What is the coefficient of determination? What does it tell you?

*The coefficient of determination is $R^2$, which is $0.6154$. This gives the 'percent linearity' of the data, i.e. the percent of the data explained by the model.*

(c) Give a 95% confidence interval for $\beta_5$. Interpret the result in the context of the problem.

*The variable $\beta_5$ is SP. From the table, we have $b_5 = 0.2907$, $SE_{b_5} = 0.0935$. Because $n = 1030$, we have degrees of freedom $n - p - 1 = 1021$. Using the $t$-table, we have $t^* = 1.962$. Therefore,*

$$b_5 \pm t^* SE_{b_5}$$
$$0.2907 \pm 1.962(0.0935)$$

*so that we have confidence interval $(0.107, 0.474)$. Therefore, we are 95% certain that the coefficient of SP is between $0.107$ and $0.474$, i.e. we are 95% certain that when SP increases by 1, the comprehensive strength goes up by an amount between $0.107$ and $0.474$.*

(d) For which variables do there seem to be a (linear) association between comprehensive strength and the given variable? For which does there not seem to be? Explain. [Use a significance of $\alpha = 0.05$.]

*For this, we could use a $t$-test on the coefficients. The table gives these values. There seems to be an association for the following variables: cement, BF, FlyAsh, Water, SP, and Age. There does not seem to be an association for CoarseAg and FineAg. This is all seen from the corresponding $p$-values on the table for the $t$-tests.*

(e) Use the data in the table to perform an $F$-test for this model. State the null and alternative hypothesis, test statistic, $p$-value, and state your conclusion. [Use $\alpha = 0.10$.]

$$\begin{cases} H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = \beta_7 = \beta_8 = 0 \\ H_a : \text{not all the } \beta_i \text{ are } 0 \end{cases}$$

*We have $F = 204.27$ with $p$-value $0.000$ (the degrees of freedom are $(8, 1021)$). Therefore, we reject the null hypothesis so that there is a relationship between the comprehensive strength of concrete and* at least one *of the given predictive variables, i.e. at least one of the coefficients is nonzero.*

(f) If one were to perform a $t$-test on one of the coefficients, what is the associated degrees of freedom? What about if one were to perform the $F$-test 'by hand', what are the degrees of freedom of the numerator and denominator?

*The degrees of freedom for the $t$-tests would be $n - p - 1 = 1030 - 8 - 1 = 1021$, the degrees of freedom of the error. For the $F$-test, the degrees of freedom of the numerator is the number of variables, $8$, and the degrees of freedom of the denominator is the degrees of freedom of the error, $1021$.*