Name: ____
MAT 222
Spring 2019
Exam 1 Review

Problem 1: Indicate whether the following statements are True (T) or False (F):

(a) ________ A confidence interval of level $P\%$ for a parameter is an interval computed from sample data by a method that has a probably $P\%$ of containing the true value for the parameter.

(b) ________ Nonresponses should always be considered a source of error/bias in a study.

(c) ________ The confidence levels and hypotheses (especially the decision of $>, <, \neq$) can/should be made once the sample average $\overline{x}$, sample proportion $\hat{p}$, etc. have been collected and found.

(d) ________ One can reduce the margin of error by increasing the level of confidence, C.

(e) ________ When the population standard deviation is known, you can choose your n to obtain a confidence interval with margin of error m by computing $\left(\frac{z^*\sigma}{m}\right)^2$ and 'rounding' up.

(f) ________ The margin of error in a confidence interval accounts for all the error in a statistical inference.

(g) ________ One should examine the sample data to determine if $\overline{x} < \mu_0$ or $\overline{x} > \mu_0$ before determining the alternative hypothesis.

(h) ________ The p-value is the probability of seeing an outcome *at least* as extreme as the one observed.

(i) ________ The smaller the p-value, the more evidence there is against the null hypothesis.

(j) ________ There is a clear boundary between significant and insignificant.

(k) ________ If something is statistically significant, then it is practically significant.

(l) ________ The "significant" in statistically significant means importance.

Problem 2: Indicate whether the following statements are True (T) or False (F):

(a) ________ Failing to reject H_0 indicates that H_0 is true.

(b) ________ A two-sided test at significance level α can be carried out directly from a confidence interval with confidence level $C = 1 - \alpha$.

(c) ________ The p-value is the smallest level α at which the data is significant.

(d) ________ Large samples will detect even tiny deviations from the null hypothesis and thus will be statistically significant.

(e) ________ The first step in data analysis is to explore the data visually, whenever possible, to help determine what analyses can be/should be performed.

(f) ________ There is no way of knowing whether your statistical test can detect a mean in advance? For example, the median American income was once X but you believe it has shifted to Y, can you determine in advance if your test is likely to detect this?

(g) ________ You cannot test a hypothesis on a data set which suggested the hypothesis.

(h) ________ Power likelihood that you will reject H_0.

(i) ________ The less likely you are to make a Type II error, the greater the power.

(j) ________ A Type I error is failing to reject a false null hypothesis.

(k) ________ The greater the chance of making a Type I error, the greater the chance of making a Type II error.

(l) ________ The farther the considered alternative mean, μ_A, is from the null hypothesis mean, μ_0, the greater the chance of making a Type II error.

(m) ________ The larger the degrees of freedom, the 'closer' the t-distribution is to a normal distribution.

Problem 3: Indicate whether the following statements are True (T) or False (F):

(a) ________ In a one-sample t-test with degrees of freedom 96, a 95% confidence interval would use $t^* = 1.984$.

(b) ________ If one were to accidentally use a z-method instead of a t-method, the final p-value would be greater than the one obtained using the correct t-method.

(c) ________ Suppose you are performing a statistical analyses with a t-test. Specifically, you are testing $H_0 : \mu = 1234$ and $H_a : \mu \neq 1234$. If you have degrees of freedom 26 and find $t = 2.373$, then your p-value is $p = 0.02$.

(d) ________ In a study comparing pre versus post results for some test, whether academic, medical, etc., one can choose between a matched pairs analysis and a two-sample t (one sample being the pre results and the other sample being the post results).

(e) ________ A lack of significant results supports the null hypothesis.

(f) ________ A robust statistical procedure is one which the statistical probability calculations do not vary much if there are 'small' violations to the assumptions made.

(g) ________ t-procedures are robust to non-normality.

(h) ________ For t-procedures, it is more important that the sample be a SRS than the population distribution be normal.

(i) ________ If there is *some* skewness in the data set and $n = 45$, then a t-procedure is inappropriate.

(j) ________ The difference between two sample means with unknown standard deviations follows *exactly* a t-distribution

(k) ________ When performing a test with unknown population standard deviations and sample sizes 36 and 65, respectively, the *only possible* dof is 35.

(l) ________ It would be appropriate to use a two-sample t-procedure, with sample sizes 45 and 36, if the data sets were skewed.

(m) ________ If you were considering a pooled t-test with sample standard deviations $s_1 = 0.035$ and $s_2 = 0.099$, then a pooled t-test could be possible.

Problem 4: Indicate whether the following statements are True (T) or False (F):

(a) ________ The pooled standard deviation is $s_p = \dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$.

(b) ________ The sample size required to obtain a confidence interval with an expected margin of error, at most m, for a population mean satisfies $m \geq t^* s^*/\sqrt{n}$, where s^* is the approximated population standard deviation.

(c) ________ A null and alternative hypotheses $H_0 : \mu_1 = \mu_2$ and $H_a : \mu_1 > \mu_2$ is the same as $H_0 : \mu_1 - \mu_2 = 0$ and $H_a : \mu_1 - \mu_2 > 0$.

(d) ________ If one uses a pooled p-procedure, the final p-value you will obtain is lower than the one you would obtain if you used a two-sample p-method.

(e) ________ To determine the sample size required to obtain a level C confidence interval with margin of error m in a p-procedure with no prior estimate of p^*, one uses $p^* = 0.50$.

(f) ________ In a p-procedure, when p is unknown, one uses test statistic $t = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$ and degrees of freedom $n-1$.

(g) ________ If you fail to reject a null hypothesis $H_0 : p_1 - p_2 = 0$, then you can conclude there is no difference in these population proportions.

(h) ________ If one wishes to estimate the difference $p_1 - p_2$ with a 90% certainty and error at most 2%, it suffices to run a study using 3,403 people.

(i) ________ The *only* way to determine if there is a difference between p_1 and p_2 is to test the difference $p_1 - p_2$.

(j) ________ In a confidence interval of level C, the shaded ends of a distribution have area $\frac{1-C}{2}$, while the 'center' has area C.

(k) ________ The corresponding z^*-values for a 90%, 95%, and 99% confidence interval are 1.645, 1.960, and 2.576, respectively.

(l) ________ The t^*-values for a 90%, 95%, and 99% confidence interval are 1.775, 2.020, and 2.879, respectively.

Problem 5: A safe driving advocacy group claims that the mean speed of drivers on a highway exceeds the posted speed limit of 65 mph. To test this, the group plans to select a SRS of 16 cars and to reject the null hypothesis that the mean speed is 65 mph if the sample mean speed is higher than 66.5 mph. The standard deviation for their data was 3 mph.

(a) Find the probability of a Type I error.

(b) What is the probability of rejecting the null hypothesis (assuming H_0 is true).

(c) Find the probability of a Type II error if the true mean is 66.5 mph.

(d) Based on your answer in the previous part, do you think this test has adequate power? Explain. How could you increase the power?

(e) What do you need to guarantee the statistical validity of the previous parts? Explain.

(f) What would change if you knew the speeds were normally distributed with standard deviation 3 mph. Redo the previous parts with this assumption.

(g) If you knew that the population standard deviation was 3 mph and that alone, could you use the method of the previous part? Explain.

Problem 6: A British study compared 98 drivers and 93 conductors of London double-decker buses regarding daily caloric consumption. Some of the study results are summarized below:

	n	$\overline{x}$	s
Drivers	98	2821	435.6
Conductors	93	2844	437.3

(a) Construct a 96% confidence interval for the difference of the mean for the caloric intake of drivers and conductors. State your conclusions. Furthermore, justify the validity of your interval.

(b) Is there significant evidence, at the 5% level, that conductors consume more calories per day than do drivers? State your hypotheses and conclusion, give your test statistic, and find a p-value. [Assume that $\sigma_{\text{Drivers}} \neq \sigma_{\text{Conductors}}$.] Is this test valid?

(c) Suppose instead one had used the pooled two-sample t-test. How would your test statistic change? Calculate s_p. What is the degrees of freedom in this test? For pooled t-tests, what are the assumptions? Finally for pooled t-tests, what group sizes result in the best results?

Problem 7: A sociologist is interested in measuring the proportion of Native Americans who are able to easily access government assistance (assuming they have made an attempt to receive services). The sociologist decides to define 'easily access' as receiving government assistance within one month of applying for assistance.

(a) How many observations should be taken to estimate, at a 90% confidence level, the population proportion within a margin of error of 0.04? What could go wrong using this many people?

(b) A random sample of 200 Native Americans at a single reservation who attempted to access government assistance was collected. Of these individuals, 155 *did not* receive assistance within a month of applying for it. Find a 95% confidence interval for the true proportion of Native Americans who are able to easily access government assistance. What are the possible mathematical sources of error? What are the possible design problems that could cause error?

Problem 8: Researchers wanted to measure the affect of alcohol on the development of the hippocampal region in adolescents. The hippocampus is the portion of the brain responsible for long-term memory storage. The researchers randomly selected 12 adolescents with alcohol use disorders. They wanted to determine whether the hippocampal volumes in the alcoholic adolescents were less than the typical volume of 9.02 cubic centimeters with a population standard deviation of 0.67. The sample had a mean of 8.10.

(a) Since the sample size is 12, what must we assume about the distribution of hippocampal volumes in alcoholic adolescents?

(b) Construct a 99% confidence interval for the average hippocampal volumes in the alcoholic adolescents.

(c) If we wanted to decrease our margin of error to 0.3, how many more people would we need to observe?

(d) Conduct a hypothesis test that the researchers conducted at $\alpha = 0.01$ significance level. Determine the null and alternative hypothesis, test statistic, p-value or critical value, and state your conclusion in the context of the problem.

Problem 9: The attention span of small children (ages 3–5) is claimed to be normally distributed with a mean of 15 minutes and a standard deviation of 4 minutes. A test is to be performed to decide if the average attention span of these children is really this short, or if it is longer. To test the hypotheses $H_0 : \mu = 15$ versus $H_a : \mu > 15$, you plan to select a SRS of 10 children who will watch a TV show they have never seen before and to record the time until they walk away from the show. Suppose that you decide to reject the null hypothesis if the sample mean is greater than 17 minutes.

(a) Find the probability of a Type I error for this test.

(b) Find the probability that you fail to reject the null hypothesis, assuming that H_0 is true.

(c) Find the probability of a Type II error if the true mean is 18 minutes. Use this to find the power of this test.

From the Journal of Psychiatry, May 2000

(d) Does this test have adequate power to detect if the true mean is 18 minutes? Explain. How could you increase the power?

(e) Assuming that this experiment is run twice, with independent samples, and that $\mu = 18$ minutes. What is the probability that *both* tests reject the null hypothesis?

(f) Explain why the results above are 'mathematically valid'. What if the population were not normally distributed, what would we need to change about the experiment?

Problem 10: Gallup conducted telephone interviews with a sample of 1,039 American adults, asking the following question: "What do you think is the most important problem facing this country today?"

(a) According to the report (November 14, 2013), 19% of the 1,039 adults in the sample mentioned healthcare as the country's most important problem. Construct a 95% confidence interval for the proportion of Americans who consider healthcare as the country's most important problem. Report your results.

(b) What problems could arise with the interval given in (a)?

(c) Suppose that 96 men among 562 men in the survey and 101 women among 477 women in the survey answered "healthcare" as the most important issue in the country. Test if the proportion of American women who consider "healthcare" as the most important problem is higher than the same proportion of American men. Use a significance level of $\alpha = 0.01$. What changes if one hypothesized that there was a 2% difference?

Problem 11: A researcher wished to compare the average amount of time spent per week in extracurricular activities by High School students in a suburban school district with that in a school district of a large city. To test the claim that the mean amounts of time spent in extracurricular activities per week are different, two sets of simple random samples were taken, and the data is summarized below:

	n	$\overline{x}$ (Hours)	s (Hours)
Suburban School District	28	6.2	2.5
City School District	25	4.6	2.8

(a) State the hypotheses to test if the mean amounts of time spent in extracurricular activities per week are not equal between the two districts.

(b) Assuming that times spent are normally distributed. Calculate the value of your test statistic.

(c) What can you say about the value of p? Specify the degrees of freedom that you used.

(d) At the 5% significance level, can you conclude that the mean amounts of time spent in extracurricular activities per week are not equal between the two districts? Why or why not?

(e) Do you need the assumption in (b)? Explain.

(f) Give some reasons why a policy maker might care about the results of these statistical analyses.

Problem 12: A university took a sample of patrons from their eating establishments and asked them about their overall dining satisfaction (rated on a scale of 1–5). The following table summarizes the results for three groups of patrons:

Category	n	$\overline{x}$	s
Student Meal Plan	185	3.44	0.883
Faculty Meal Plan	35	4.04	0.983
Student No-meal Plan	116	3.47	0.764

(a) Is it reasonable to use a pooled t-test to compare the student-meal plan average with the faculty meal plan average? What about the student no-meal plan with the student-meal plan? Explain.

(b) Suppose one uses a pooled t-test to compare the faculty meal plan average with the student no-meal plan average. Construct a 95% confidence interval for the difference.

(c) Conduct an appropriate hypothesis test at $\alpha = 0.05$ to compare the faculty meal plan average with the student no-meal plan average. Report your difference estimate, degrees of freedom, test statistic, p-value, and conclusion (in the context of the problem).

(d) Are the statistical methods applied above valid? Justify your answer completely.

(e) How could one improve the results above? Explain.

Problem 13: When we conduct a test for a population mean, we often use a t-test based on a t-distribution instead of a z-test based on a normal distribution. Which of the following is the most appropriate explanation for this?

(a) The sample does not follow a normal distribution.

(b) The population standard deviation is unknown.

(c) The sample size is larger than 30.

(d) The sample size is less than 30.

Problem 14: Suppose that you obtained the p-value of 0.085 when you conducted a t-test of $H_0 : \mu = 0$ against $H_a : \mu \neq 0$. If you construct a confidence interval for μ using the same data, among the following, which is the smallest level of confidence for which the confidence interval contains 0?

(a) 90% (b) 92% (c) 95% (d) 99%

Problem 15: Consider the test $H_0 : \mu = 450$ against the alternative hypothesis $H_a : \mu \neq 450$ at the $\alpha = 0.01$ level of significance. Assume that the population standard deviation is $\sigma = 100$ and a simple random sample of size 400 is to be taken.

(a) Calculate the critical value. For what values of the sample mean will H_0 be rejected?

(b) Calculate the probability of a Type II error against the alternative $\mu = 460$.

(c) Calculate the power of the test against the alternative $\mu = 460$.

Problem 16: In a recent CBS News poll, 627 of the 1307 respondents said that the Federal government was responsible for insuring that every qualified person gets a college education.

(a) Construct a 99% confidence interval for the proportion of Americans who would agree with the above statement.

(b) Based on the confidence interval, can you say that the true population is not significantly different from 50%? Explain.

(c) Conduct a hypothesis testing for $H_0 : p = 0.5$ against $H_a : p \neq 0.5$, where p is the proportion of Americans who would agree with the above statement in (a). Use $\alpha = 0.01$. Do you get the same conclusion as in (b)?

(d) If CBS wanted to estimate p with 99% confidence and an error of at most 0.1%, at least how many people would they need to use? Does this seem feasible? What is they anticipated 46% of Americans agreeing with the statement above, at least how many people should they use?

Problem 17: A mental health counselor is interested in various stress inducers for college students at their university. One of the largest sources of stress for students is lack of sleep. Students who average low levels of sleep tend to have higher rates of mental health issues. Students should be getting 8 hours of sleep a night, on average, in order to function properly. The counselor takes a SRS of 127 college students and find an average nightly sleep rate of 6.5 hours with standard deviation 1.25 hours.

(a) Construct a 99% confidence interval for the average nightly sleep amounts students at this university have.

(b) Do students on this university get an average of 8 hours of sleep per night? State appropriate hypotheses and test them using a significance level of $\alpha = 0.01$. Be sure to state your test statistic, p-value, and conclusions in the context of the problem.

(c) What assumptions need to be made about the sample for the above methods to hold?

(d) What if the counselor wanted to measure the average number of hours of sleep per night at this university, with 30 minute accuracy. At least how many students would the sample need to include? Is this feasible. [Still assume a 99% confidence level.]

Problem 18: A test question is considered 'good' if it differentiates between prepared and unprepared students. The first question on a test was answered correctly by 62 of 80 prepared students and 26 of 50 unprepared students. Perform a significance test for the null hypothesis $H_0 : p_1 = p_2$ against $H_a : p_1 > p_2$, where p_1 is the population proportion of prepared students and p_2 is the population proportion of unprepared students. What can you conclude?

Problem 19: Particle size is an important property of latex paint and is varied to produce different finishes corresponding to different mean particle sizes. After adjusting the machine, particle size samples must be taken to determine the mean particle size being produced in order to grade the paint. The particles are normally distributed with standard deviation 100 angstroms.

(a) Find the sample size necessary for a 95% confidence interval for the mean particle size with margin of error less than 100 angstroms.

(b) Suppose you measure particles from a simple random sample of size 50, and determine the sample mean to be $\overline{x} = 2650$. Find a 95% confidence interval for the mean particle size μ.

(c) Assuming the values in the previous part, test the null hypothesis $H_0 : \mu = 2500$ versus $H_a : \mu \neq 2500$ at the $\alpha = 0.05$. Report your test statistic, p-value, and conclusions. Check your answer by using the previous part.

(d) What assumptions are required on your data for the methods you used above to be valid?

Problem 20: Radon levels in houses vary greatly by location. Samples are taken from houses in Syracuse and Ithaca with the following results: in 10 randomly selected houses in Syracuse, the average Radon level was 10.2 with standard deviation 3.4. In 8 randomly selected houses in Ithaca, the average was 13.1 with standard deviation 4.1. We test whether the Radon levels in Ithaca is higher than those in Syracuse.

(a) State suitable null and alternative hypotheses.

(b) Is there significant evidence of unequal population means at the 1% level?

(c) Is there significant evidence at the 1% level that Radon levels in Ithaca are higher than those in Syracuse?

(d) Construct a 95% confidence interval for the difference in Radon levels between Ithaca and Syracuse.

(e) What assumptions do you need on these populations for these methods to give you accurate results?

(f) Is a pooled t-test possible for this data? If yes, redo the above using pooled procedures. If not, explain why.

Problem 21: Two samples of 2,065 males and 2,235 females were obtained in order to compare the percentage of males who smoked to the percentage of females who smoked. Of the males sampled, 580 were cigarette smokers, and of the females sampled, 525 were cigarette smokers. Does the data provide sufficient evidence to conclude that the percentage of males who smoked cigarettes exceeds the percentage of females who smoke cigarettes?

(a) State the null and alternative hypotheses.

(b) Calculate the test statistic.

(c) Is the test significant at the 1% level?

(d) If the data was collected mostly by surveying couples, are the methods above valid? Explain. What other requirements do you need for these samples?

Problem 22: Some books state the IQ of Americans aged 20–34 has a mean of 110, while other books state the mean IQ is 100. Suppose that, to settle this question, 150 Americans aged 20–34 are chosen at random, and their IQ's are measured. Suppose you test

$$\begin{cases} H_0 : \mu = 100 \\ H_a : \mu > 100 \end{cases}$$

Assume the population standard deviation is equal to 30. Reject the null hypothesis if the sample mean, $\overline{x}$, is greater than 105.

(a) Find the level of significance of this test.

(b) Find the probability of a Type II error if $\mu = 110$.

(c) Find the power for this test. Is this sufficient? Explain.