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MAT 222
Spring 2019
Chapter 6: Power

"Birthdays are good for you. Statistics show that people who have the most live the longest."
–Father Larry Lorenzoni

Problem 1: Suppose you are performing a hypothesis test, given below, using a dataset with 36 data values and a standard deviation 120. To perform the test, you use a significance level of 0.05

$$\begin{cases} H_0 : \mu = 1000 \\ H_a : \mu > 1000 \end{cases}$$

(a) What is the probability of a Type I error?

(b) What is the probability that you do not reject the null hypothesis? [Assume H_0 true.]

(c) What is the probability of a Type II error if $\mu = 1020$?

(d) What is the power of the test if $\mu = 1020$?

(e) Suppose that $\mu = 1020$, and two different researchers perform a test on a SRS of 36 individuals from this population. What is the probability that both experiments determine that $\mu \neq 1000$?

Solution.

(a) $P(\text{Type I Error}) = \alpha = 0.05$.

(b) $P(\text{Do Not Reject} \mid H_0 \text{ True}) = 1 - \alpha = 0.95$. *[The question*

(c) A Type II error is when we fail to reject a false null hypothesis. We reject for $\overline{x}$ *greater than 1000 with probability* 0.05 *of occurring, i.e. with* z*-value at least* 1.645*. But then* $1.645 = \frac{\overline{x}-1000}{120/\sqrt{36}}$ *so that we reject when* $\overline{x} \geq 1032.9$*. Therefore, we fail to reject when* $\overline{x} < 1032.9$*. If* $\mu = 1020$*,*

$$z = \frac{1032.9 - 1020}{120/\sqrt{36}} = \frac{12.9}{20} = 0.645 \rightsquigarrow \frac{0.7389 + 0.7422}{2} = 0.74055$$

so that this happens with probability 0.74055. Therefore, $P(\text{Type II Error}) = 0.74055$.

(d) We know Power $= 1 - P(\text{Type II Error}) = 1 - 0.74055 = 0.25945$.

(e) Assuming they are testing the hypothesis given, the probability that they reject is 0.25945 *so that the probability that they* both *reject is a* $0.25945 \cdot 0.25945 = 0.0673 = 6.73\%$ *chance.*

Problem 2: A nationwide poll in California showed that there is a 4.2% unemployment rate. The mayor of a city in California is trying to show that their city's unemployment rate is higher to receive more state funding to alleviate the issue. They plan on taking a sample to see if their unemployment rate is higher than the statewide 4.2%. Let p be the proportion of people unemployed in the city. The hypothesis used is:

$$\begin{cases} H_0 : p = 0.042 \\ H_a : p > 0.042 \end{cases}$$

Which of the following are correct decisions? Which of the following would be a Type I error? Which of the following would be a Type II error?

(a) They conclude the unemployment rate is greater than 4.2% when it is.

(b) They conclude the unemployment rate is greater than 4.2% when it is not.

(c) They conclude the unemployment rate is not greater than 4.2% when it is.

(d) They conclude the unemployment rate is not greater than 4.2% when it is not.

Solution. *(a) Correct Decision. (b) Type I Error. (c) Type II Error. (d) Correct Decision.*

Problem 3: A university is surveying their students to see if there is support for a change in the campus' fraternity policies. They will consider making changes if there is at least a 60% approval for making changes. Let p be the proportion of students in support of changes. The hypothesis used are:

$$\begin{cases} H_0 : p \geq 0.60 \\ H_a : p < 0.60 \end{cases}$$

Which of the following are correct decisions? Which of the following would be a Type I error? Which of the following would be a Type II error?

(a) They do make changes when there is enough support.

(b) They do make changes when there is not enough support.

(c) They do not make changes when there is not enough support.

(d) They do not make changes when there is enough support.

Solution. *(a) Correct Decision. (b) Type II Error. (c) Correct Decision. (d) Type I Error.*

Problem 4: What are the ways that you can increase the power of a test? What are the ways you can decrease the probability of a Type II error?

Solution. *You can increase the number of samples taken, n, you can decrease the standard deviation, σ, or increase the significance level, α (this corresponds to decreasing z^*). Since the probability of a Type II error is the compliment of power, you would need to do the opposite: decrease n, increase σ, or decrease the significance level α (this corresponds to increasing z^*).*

Problem 5: Which of the following is a Type I error? Which of the following is a Type II error?

(a) True positive

(b) True negative

(c) False positive

(d) False negative

Solution. *Type I error is a false positive (c) while a Type II error is a false negative (d).*

Problem 6: A consumer protection bureau is investigating whether a supplement producing company is misleading consumers about the chemical makeup of their supplements, i.e. the percentage listed is not the percentage in the pills. The amount listed on the product is 50 mg. The bureau will test a collection of 9 large pallets of pills from the company, finding the average amount of the supplement in each of the pallets. They will reject the claim that 50 mg of product is in the drug if their sample mean is less than 40 mg. Assume the amount of supplement is normally distributed with standard deviation 11.7 mg. What is the probability of a Type II error if the true mean is 48 mg? What is the corresponding α level for their hypothesis test?

Solution. *We reject if $\overline{x} < 40$, so if $\mu = 48$, then we calculate the probability to be*

$$z = \frac{40 - 48}{11.7/\sqrt{9}} = \frac{-8}{3.9} = -2.05 \rightsquigarrow 0.0202.$$

The power is therefore 0.0202. Then the probability of a Type II error must be $1 - 0.0202 = 0.9798$. To find α, we know we reject if $\overline{x} < 40$. If $\overline{x} = 40$, then this has probability

$$z = \frac{40 - 50}{11.7/\sqrt{9}} = \frac{-10}{3.9} = -2.56 \rightsquigarrow 0.0052.$$

Therefore, we have $\alpha = 0.0052$.

Problem 7: Consider the situation in the previous problem. If the sample size is increased, keeping everything else the same, then

(a) the probability of a Type I error increases.

(b) the probability of a Type II error decreases.

(c) the significance level decreases.

(d) the probability of failing to reject H_0 when H_0 is true decreases.

Solution. *(b)*

Problem 8: Suppose one performs a hypothesis test of $H_0 : \mu \geq 30$ against $H_a : \mu < 30$ using a significance of $\alpha = 0.05$ in a dataset where $n = 100$ and $\sigma = 100$. What is the probability of a Type I error? Assume that $\mu = 26$. What is the power of the test? What is the probability of a Type II error?

Solution. $P($*Type I Error*$) = 0.05$. *We know reject* H_0 *for* $\overline{x}$ *with their* z*-value less than* -2.575. *But then* $-2.575 = \frac{\overline{x}-30}{100/\sqrt{100}}$ *so that* $\overline{x} \leq 4.25$. *But if* $\mu = 26$, *this happens with probability*

$$z = \frac{4.25 - 26}{100/\sqrt{10}} = -0.69 \rightsquigarrow 0.2451.$$

Therefore, we have power 0.2451 *so that the probability of a Type II error is* $1 - 0.2451 = 0.7549$.